MATH 20000: Topics on Hour Test 2

Math 200: Section 11, Autumn 2015, Instructor: William Feldman

The test is cumulative but the main focus will be on the topics covered since this first hour test. You should be familiar with the concepts covered in lecture and in the textbook from Chapters 4, 5, 6.1-6.2, and 7.1-7.2 . Here is a list of concepts/techniques that you should be familiar with, as usual it is not exhaustive but rather a suggestive guide.

- **General Advice**
  - Make your drawings very carefully, make them large enough to see the important details, clearly label the axes, clearly label several important points which help to interpret the scale.
  - Review what the domain and range of a function are if you are not clear on this. Understand the difference between a scalar valued function and a vector field. I am especially interested in checking that your basic understanding of the properties of the objects we are studying (scalars, vectors, and to a lesser degree matrices) is sound.
- **Chapter 4**
  - Velocity, acceleration of a path.
  - Differentiation rules for paths.
  - Checking whether a path solves a differential equation like Newton's Law $ma(t) = mc''(t) = F(c(t))$
  - Arc length of a path.
  - Vector fields, gradient vector fields, flow lines of vector fields (drawing the vector field, drawing the flow lines, checking whether a path c is a flow line of a vector field V).
  - Divergence of a vector field, curl of a vector field. How to compute them, what kind of functions are they (scalar, vector), basic facts like gradients have $\nabla \times (\nabla f) = 0$ for f scalar valued function on $\mathbb{R}^3$ and $\nabla \cdot (\nabla \times F) = 0$ for $F : \mathbb{R}^3 \to \mathbb{R}^3$ a vector field.
- **Chapter 5**
  - Cavalieri's principle, reducing integral over a rectangle to iterated integrals.
  - Fubini's theorem (being able to apply it).
  - Computing integrals over elementary regions in $\mathbb{R}^2$ and $\mathbb{R}^3$, changing the order of integration. You should be able to interpret in all directions between a description of the domain in words (e.g. the region of $x^2 + y^2 + z^2 \leq 1$ with $z \geq 0$), an iterated integral over the domain, a description of the domain as an elementary domain in set theoretic notation.
  - Tricks to compute integrals over non-elementary regions like splitting up into several elementary regions and summing.
- **Chapter 6**
  - Definitions of one-to-one, onto and invertible. Definition of the image of a set under a mapping.
  - Images of linear maps (parallelograms are mapped to parallelograms), computed the area of a parallelogram (or parallelopiped) by the determinant of a matrix/linear map.
  - The jacobian determinant of a transformation.
  - The change of variables theorem, computing integrals using linear changes of variables, polar, cylindrical and spherical coordinates.
  - Expressing domains of $\mathbb{R}^3$ in different coordinate systems (closely related with computing integrals using change of variables).
- **Chapter 7**
  - The path integral, the line integral (what is the difference with path integral?). Computing these in various examples.
  - Reparametrizations of paths and how the line/path integral can change under reparametrization.
  - Line integrals of gradient vector fields.
  - Line integrals over simple curves and over curves with several components.