MATH 20000: Topics on Hour Test I

Math 200: Section 11, Autumn 2015, Instructor: William Feldman

You should be familiar with the concepts covered in lecture and in the textbook from Chapters 1, 2 and 3.1-3.4. Here is a list of concepts/techniques that you should be familiar with.

- **Chapter 1**
  - Vector operations (addition, subtraction, scalar multiplication) and their geometric interpretation.
  - The parametric equation of a line given (1) point and vector, (2) two points.
  - The inner product, the norm, their geometric interpretation.
  - Orthogonal projection.
  - Determinants of 2×2 and 3×3 matrices and their geometric interpretation (as area/volume of parallelogram and parallelopiped respectively).
  - Cross product, geometric interpretation, algebraic rules.
  - Triple product ($(a \times b) \cdot c$) relation with determinants of 3×3 matrices.
  - Equation of a plane in $\mathbb{R}^3$ given point and normal vector.
  - Distance from a point to a plane.
  - Definition of cylindrical and spherical coordinate systems and basic geometry, converting between coordinate systems.
  - Inner product in $\mathbb{R}^n$, matrix multiplication and matrix vector multiplication.
- **Chapter 2**
  - The definition of a vector valued function, its domain and range.
  - The graph of a function, level sets, sections. Using the level sets and sections to produce an accurate drawing of the graph.
  - Open sets, definition of limits by neighborhoods.
  - Basic properties of limits.
  - Definition of continuity, basic properties, checking whether a function is continuous.
  - The partial derivative of a function of multiple variables.
  - Definition of differentiability for vector valued functions of several variables (i.e. $f : \mathbb{R}^n \to \mathbb{R}^m$), calculating the matrix Df. What is a C^1 function on a domain $U \subset \mathbb{R}^n$.
  - The tangent plane to the graph of a function $f : \mathbb{R}^n \to \mathbb{R}$.
  - Relationship between differentiability and continuity.
  - Paths/curves in $\mathbb{R}^n$ (what is the distinction?). Parametrizations, re-parametrizations of a curve by different paths.
  - Velocity vector of a path, speed of a path, tangent vector/line to a curve.
  - Differentiation rules, products, sums, constant multiples, quotients, and the chain rule.
  - Directional derivatives.
  - Understanding the gradient ∇f as direction of fastest increase, normal to level surfaces of f.
  - The equation of the tangent plane to a level surface of $f : \mathbb{R}^n \to \mathbb{R}$.
- **Chapter 3**
  - What is a C^2 function, the mixed partial derivatives, equality of mixed partials.
  - Taylor's Theorem up to second order for functions of several variables and its interpretation.
  - Local minima, maxima, critical points.
  - First derivative test for local extrema.
  - Quadratic functions in $\mathbb{R}^n$ and the Hessian of $f : \mathbb{R}^n \to \mathbb{R}$.
  - Positive/negative definiteness for a quadratic.
  - The second derivative test for local extrema, esp. the case $n = 2$ where it is much easier to check positive-definiteness.
  - Saddle points.
  - Strategy for finding the global maxima and minima of $f : U \subseteq \mathbb{R}^n \to \mathbb{R}$ by parametrizing ∂U.
  - The method of Lagrange multipliers for constrained extrema.
  - The method of Lagrange multiplies for finding the global maxima and minima of $f : U \subseteq \mathbb{R}^n \to \mathbb{R}$.