Practice Questions for Final Exam

Math 200: Section 11, Autumn 2015

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On the exam you will be asked to always show the work leading to your answer for full credit. The actual exam will have approximately eight questions. I have not written questions here about all the possible topics which may be on the exam, refer to your old exams and practice problems as well as the topics list and your previous homework assignments for more possible material.

1. Compute the volume of the region between the cone $z = \sqrt{x^2 + y^2}$ and the parabola $z = x^2 + y^2$ in the annular cylinder $1/2 \leq \sqrt{x^2 + y^2} \leq 1$.
2. Change the order of integration in $\int_1^2 \int_{-\log x}^{\log x} f(x, y) dy\ dx$.
3. (Chapter 3.4 #21) Design a cylindrical can (with a lid) with volume 1 and with minimal surface area.
4. Compute the total flux through the surface $S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1, z \geq 1/\sqrt{2}\}$ (a spherical cap) of the constant vector field $F(x, y, z) = (0, 0, 1)$.
5. Let D be a simple region in $\mathbb{R}^2$. Use Green's Theorem (in the form of the divergence theorem Chapter 8.1 Thm 4) to show that $2\text{Area(D)} = \int_{\partial D} (x, y) \cdot \vec{n}\ ds$ where $\vec{n} : \partial D \to \mathbb{R}^2$ is the unit normal to ∂D.
6. Again using Green's Theorem one can show that for a simple region $D \subset \mathbb{R}^2$ with boundary ∂D_+ oriented counter-clockwise $2\text{Area}(D) = \int_{\partial D_+} xdy - ydx$. Use this formula to compute the volume of the ellipse $D = \{(x, y) \in \mathbb{R}^2 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\}$.
7. Check that $T(u, v) = (4u + v, u + 3v)$ maps the unit square $D_* = [0, 1] \times [0, 1]$ one-to-one and onto the parallelogram D generated by the vectors $(4, 1)$ and $(1, 3)$, $D = \{(x, y) \in \mathbb{R}^2 : (x, y) = t(4, 1) + s(1, 3)$ with $0 \leq t, s \leq 1\}$. Use the change of variables theorem with the mapping T to compute
$$\int \int_D xy\ dxdy.$$
8. You are hiking on a mountain with height profile $h(x, y) = 1 - x^2 - y^2$ and you are currently standing at the point $(1, 1)$. What is the equation of the tangent line L to the level set $\{h(x, y) = -1\}$ at the point $(1, 1)$. What will be $\frac{d}{dt} h(c(t))\big|_{t=0}$ if you travel along a path $c(t)$ with $c(0) = (1, 1)$ and c is parallel to the tangent line L? Suppose that instead you travel along the path $\gamma(t) = (1, 1) + tv$ for some unit vector $v \in \mathbb{R}^2$. Express your height as a function of t. How can you choose v so that you are climbing with a 3% grade at $t = 0$ (tangent of angle with the horizontal is .03) there are two possible choices.
9. Review the definitions of domain and range of a function especially in the context of the problems I have given you about divergence, curl, composition of functions, derivatives of vector valued functions etc.