Practice Questions for Hour Test 1
Math 200: Section 11, Autumn 2015
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On the exam you will be asked to always show the work leading to your answer for full credit. This practice exam is not designed with the time limit in mind, I just wanted to give you lots of practice questions to work on.

Problem 1. Let P be the plane in $\mathbb{R}^3$ with normal vector $v = (1, 1, -1)$ and containing the point $x = (1, 0, 0)$. Let $y = (2, 0, 0)$,

(i) Find the distance of the point y from the plane P.
(ii) Find the point z in P so that $\|x - z\|$ is equal to the minimal distance calculated in the previous part.
(iii) Write the equation for the line from x to z.

Problem 2. Define $f : \mathbb{R}^2 \to \mathbb{R}^2$ by $f(x, y) = (xy, x^2 - y^2)$

(i) Compute the derivative of f, Df, which is a 2×2 matrix.
(ii) Write down the first order Taylor expansion of f at the point $(1, 1)$.
(iii) Let $g : \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function. What are the domain and range of the composition of functions $h = g \circ f$? What are the dimensions of the matrix Dh?
(iv) Compute Dh in terms of the partial derivatives of g, $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$.

Problem 3. Let $C = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$:

(i) Draw C.
(ii) Find a parametrization of C by a path $c(t)$, $c : \mathbb{R} \to \mathbb{R}^2$, so that $c(0) = (0, 1)$ and the speed of the path is always equal to 2.
(iii) Let $f(x, y) = x^2 + y^2$, what is the value of $\nabla f(0, 1) \cdot c'(0)$ and why?
(iv) Let $g(x, y) = x + 2y$, compute the minimum and maximum values of g on C, using (a) the parametrization c, (b) the method of Lagrange multipliers.

Problem 4. The following are statements are either True or False. If you answer False give an example (as simple as you can) which shows that the statement is False.

(i) Let $f : \mathbb{R}^2 \to \mathbb{R}$. If $\frac{\partial f}{\partial x}(0, 0)$ and $\frac{\partial f}{\partial y}(0, 0)$ exist then all the directional derivatives $\frac{d}{dt} f(x + tv)|_{t=0}$ for $v \in \mathbb{R}^2 \setminus \{(0, 0)\}$ exist as well.
(ii) If a function $f : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at $x_0 \in \mathbb{R}^n$ then it is continuous at x_0 as well.
(iii) If $D \subset \mathbb{R}^n$ is closed then every continuous function $f : D \to \mathbb{R}$ attains its absolute max and min.

Problem 5. Let R be a rectangular parallelopiped with edges of length a, b, c. For example the paralellopiped spanned by the vectors $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ in $\mathbb{R}^3$.

(i) Let $V(a, b, c)$ be the volume of the paralellopiped, write a formula for V.
(ii) Let $L(a, b, c)$ be the total length of the edges of R, write a formula for L.
(iii) Let $S(a, b, c)$ be the total surface area of the faces of R, write a formula for S.
(iv) For what values of a, b, c is the volume of the paralellopiped maximized under the constraint that the total length of the edges is 1, i.e. $L(a, b, c) = 1$? What is this maximal volume?
(v) For what values of a, b, c is the volume maximized under the constraint that the surface area $S(a, b, c) = 1$? What is the maximal volume in this case?