MATH 6220 HOMEWORK 6

DUE DATE: N/A

This homework does not need to be turned in, although the material could appear on the exam. This is an initial version more problems may be added.

Problems:

1. Apply Picard’s little theorem to make a conclusion about meromorphic functions $f : \mathbb{C} \to \mathbb{C}^\ast$.
2. What are the possible ranges of holomorphic functions $f : \mathbb{C} \setminus \{0\} \to \mathbb{C}$?
3. (Assumptions of Montel’s theorem)
   (a) Show that there is a sequence of entire function $f_n$ so that $0 \not\in f_n(\mathbb{C})$ for all $n$ but $f_n$ have no subsequence converging locally uniformly to a holomorphic $f$ or $\infty$.
   (b) Show that there is a sequence of entire function $f_n$ so that $\{0,1\} \not\in f_n(\mathbb{C})$ for all $n$ but there is no subsequence converging locally uniformly on $\mathbb{C}$ to a holomorphic function.
4. Show that if $f : \mathbb{C} \to \mathbb{C}^\ast$ is meromorphic and the cardinality of $f^{-1}(\zeta)$ is at most $n$ for all $\zeta \in \mathbb{C}$ then $f$ is rational. [Note: you can do it without the Picard theorems]