MATH 6220 HOMEWORK 5

DUE DATE: MONDAY, APRIL 18

Final version: If you do not have access to the textbook for problems let me know and I can help.

Computations:

1. Give a domain larger than $\mathbb{D}$ on which the function element $(\sum_{k=0}^{\infty} z^k, \mathbb{D})$ admits unrestricted analytic continuation (and prove it).
2. Let $\Delta_0 = D_1(1)$ and $f_0(z) = z^{1/2}$ be the principal branch of the square root on Δ_0 (i.e. $\text{Arg}(z) \in (-\pi, \pi]$). Let $\gamma(t) = e^{2\pi i t}$ and $\eta(t) = e^{4\pi i t}$ for $t \in [0,1]$. Find an analytic continuation (f_t, Δ_t) of f_0 along γ and show that $[f_1]_1 = [-f_0]_1$. Find an analytic continuation $(\tilde{f}_t, \tilde{\Delta}_t)$ along η and show that $[\tilde{f}_1]_1 = [f_0]_1$.

Problems:

1. Stein and Shakarchi, Chapter 5, Exercises 11, 13, 14, 15.
2. Let $\gamma : [0,1] \to \mathbb{C}$ be a path and (g_t, Δ_t) be an analytic continuation along γ. Suppose Ω is a region so that $g_t(\Delta_t) \subset \Omega$ for all t, and there is an analytic function $f : \Omega \to \mathbb{C}$ so that $f(g_0(z)) = z$ on Δ_0. Show that $f(g_t(z)) = z$ for all $z \in \Delta_t$ for all $t \in [0,1]$. **Hint:** Show that $T = \{t \in [0,1] : f(g_t(z)) = z \text{ on } \Delta_t\}$ is both open and closed in $[0,1]$.
3. Let Ω be a domain (open, connected) in $\mathbb{C}$ and $f : \Omega \to \mathbb{C}$ holomorphic. Let $Z = \{z \in \Omega : f'(z) = 0\}$. For each $w_0 \in f(\Omega) \setminus f(Z)$ show that there is a function element (g, Δ) with $w_0 \in \Delta$ so that $(f \circ g, \Delta)$ is the identity map on Δ. Show that if w_0 is not in the closure K of $f(Z)$ then such (g, Δ) admits unrestricted continuation on $f(\Omega) \setminus K$.
4. Stein and Shakarchi, Chapter 9, Exercise 1.