MATH 6220 HOMEWORK 4

DUE DATE: MONDAY, APRIL 4

FINAL VERSION If you do not have access to the textbook for problems let me know and I can help.

Computations:

1. Stein and Shakarchi, Chapter 8, Exercise 4, 8

Problems:

1. Stein and Shakarchi, Chapter 8, Exercises 12
2. Show that if $\Omega \subset \mathbb{C}$ is a simply connected domain and $a, b \in \Omega$ and $a, b \in \Omega$ then there is a holomorphic bijection $f : \Omega \to \Omega$ so that $f(a) = b$ and $f(b) = a$.
3. (Symmetries) Suppose that $\Omega \subset \mathbb{C}$ is a simply connected domain proper with $0 \in \Omega$ and Ω has a rotational symmetry $e^{i\theta_0}\Omega = \Omega$. Show that if f is a conformal mapping from Ω to $\mathbb{D}$ with $f(0) = 0$ then
$$f(e^{i\theta_0} z) = e^{i\theta_0} f(z).$$
4. Let $f : \mathbb{D} \to \mathbb{D}$ call $f_1(z) = f(z)$ and define inductively $f_{n+1} = f \circ f_n$. Show that f_n has a subsequence converging locally uniformly on $\mathbb{D}$. Suppose that the sequence $f_n \to f_\infty$ converges pointwise on $\mathbb{D}$, what are the possible limit functions f_∞?
5. Stein and Shakarchi, Chapter 5, Exercises 1,2,7,9.