MATH 6220 HOMEWORK 1

DUE DATE: ~~FRIDAY, JANUARY 28~~ MONDAY, JANUARY 31

Final version. If you do not have access to the textbook for problems let me know and I can help.

Computations:

1. Find a number z so that $z^2 = i$. Write it in the form $z = x + iy$.
2. Stein & Shakarchi Chapter 1: 1,9,16(abcd), 25

Problems:

1. Stein & Shakarchi Chapter 1: 3, 4, 5, 7 (in part (b), denoting F_w the function from the problem, show also that F_{-w} is the inverse of F_w **[Correction]** F_w **is its own inverse I misread the definition in the book**), 23, 24
2. Prove that if $f : \mathbb{C} \to \mathbb{C}$ is C^2, viewed as a function $\mathbb{R}^2 \to \mathbb{R}^2$, and f is holomorphic on $\mathbb{C}$ then f' is holomorphic.
3. Let γ be a smooth curve in $\mathbb{C}$ and f continuous, when does $\overline{\int_\gamma f(z)dz} = \int_\gamma \overline{f(z)}dz$? **[Update] answer the following reworded question:** Let γ be a smooth curve in $\mathbb{C}$ when does $\overline{\int_\gamma f(z)dz} = \int_\gamma \overline{f(z)}dz$ for <u>all</u> f continuous?
4. Suppose that $f : \mathbb{D} \to \mathbb{C}$ is holomorphic and $|f'(z)| \leq M$ then $|f(z) - f(0)| \leq M|z|$. Formulate and prove an analogous bound for $|f(z) - f(w)|$ when $f : \Omega \to \mathbb{C}$ where Ω is a general path-connected domain.
5. Show that if f is holomorphic on an open connected set $\Omega \subset \mathbb{C}$ and $f'(z) = 0$ in Ω then f is constant. What if Ω is not connected?
6. Show that there is a holomorphic function on $\mathbb{C} \setminus (-\infty, 0]$ which is equal to $z^{1/2}$ on the positive reals. Hint: use the result of Stein & Shakarchi Chapter 1 problem 9.