Abstracts for
Markov Processes and Related Topics
July 10–13, 2006

July 6, 2006

1 Invited Talks:

1. **Anna AMIRDJANOVA**, University of Michigan

   Properties and Applications of Multiple Stochastic Fractional Integrals

   The talk focuses on the properties of multiple stochastic integrals defined with respect to general Gaussian processes, but with special attention given to multiple stochastic integrals with respect to a persistent fractional Brownian motion (fBm). Applications of multiple stochastic fractional integration to the problem of nonlinear filtering with fBm observation noise will be discussed. If time permits, some links between multiple stochastic integrals and stochastic path integrals will also be presented.

2. **Søren ASMUSSEN**, Aarhus University, Denmark

   Some Applications of the Kella–Whitt Martingale

   Kella & Whitt (JAP, 1992) introduced a class of martingales for processes of the form $Z_t = X_t + Y_t$ where X_t is a Lévy process and Y_t an added adapted component. The martingales are basically certain stochastic integrals w.r.t. the classical exponential Wald martingales. We outline some applications to problems in finance and queueing, e.g. passage problems for many-server queues, finite buffer models (two-sided reflected Lévy processes), and price calculations for certain in options and swaps. Also a Markov additive extension is outlined.

3. **Donald DAWSON**, Carleton University, Ottawa, Canada.

   Catalytic Branching Processes

   A catalytic branching diffusion processes is a continuous state branching process in which the branching rate depends on the presence of a catalyst. A catalytic branching network corresponds to a multitype system in which some types serve as catalysts for other types. Catalytic branching networks in which there are closed cycles of catalytic types and catalytic systems distributed in space pose a number of challenging mathematical problems. In this lecture we give a review of different aspects of this class of processes and discuss recent joint work with Ed Perkins, and with Andreas Greven and Iljana Zahle.

4. **Eugene DYNKIN**, Cornell University

An Application of Markov Processes to Nonlinear Analysis

The Martin boundary theory allows one to describe all positive solutions of a linear elliptic equation $Lu = 0$ in an arbitrary domain E of a Euclidean space $\mathbb{R}^d$. A probabilistic version of this theory (developed by Doob, Hunt, Kunita–Watanabe and others) is based on applying a diffusion with the generator L. Our goal is to describe the set U of all positive solutions of a semilinear equation $Lu = \psi(u)$ where ψ is a positive valued function on the half-line $[0, \infty)$. For a class of functions ψ the operator $Lu - \psi(u)$ is intimately related to a branching measure-valued Markov process which we call an (L, ψ)-superdiffusion. We start from a class $U_1 \subset U$ of solutions that are dominated by solutions of the linear equation $Lu = 0$. We call elements of U_1 moderate solutions. A solution u is called σ-moderate if it is the limit of an increasing sequence of moderate solutions. We describe all σ-moderate solutions in terms of (L, ψ)-superdiffusion. We conjectured that all solutions are σ-moderate. This conjecture was proved by Mselati in the case of the Laplacian L and $\psi(u) = u^2$. [He used the Brownian snake—a path-valued process introduced by Le Gall.] Later his result was extended by Dynkin and Kuznetsov to a general L and $\psi(u) = u^\alpha$ with $1 < \alpha \leq 2$.

5. **Pat FITZSIMMONS**, University of California at San Diego

Recurrent Extensions of Self-Similar Markov Processes

Let X be a self-similar Markov process with values in the non-negative half line, such that the state 0 is a trap. We present a necessary and sufficient condition for the existence of a self-similar recurrent extension of X that leaves 0 continuously. This condition is expressed in terms of the Lévy process associated with X by the Lamperti transformation.

6. **Wendell H. FLEMING**, Brown University

Risk Sensitive Stochastic Control and Differential Games

We gives a concise introduction to risk sensitive control of Markov diffusion processes and related two-controller, zero-sum differential games. The method of dynamic programming for risk sensitive control problem leads to a nonlinear partial differential equation of Hamilton–Jacobi–Bellman type. In the totally risk sensitive limit, this becomes the Isaacs equation for the differential game. There is another interpretation of the differential game using the Maslov idempotent probability calculus. We call this a max-plus stochastic control problem. These risk sensitive control/differential game methods are applied to problems of importance sampling for Markov diffusions.

7. **Peter GLYNN**, Stanford University

Numerical Schemes for Simulating SDEs under the Total Variation Norm

Discretization error for simulation of SDEs has been widely studied when the error is measured in the "weak" or "strong" sense. The weak error describes the error associated with computing expectations of smooth functions of the state at time t, whereas the strong error is intended to measure the quality of the discretization for path functionals of the SDE. However, the strong error only guarantees rates of convergence for the error associated with path functionals that are suitably continuous. Furthermore, the strong error has the theoretically undesirable property that it is computed in terms of a very specific "coupling" between the solution to the SDE and the approximating process. To overcome these difficulties, we consider instead the total variation distance between the probability distribution of the solution to the SDE and that of the approximation (restricted to the filtration associated with the processes observed at the discretization epochs). In this norm, it turns out that the Euler method does not converge. We therefore propose a new approximation scheme that computes solutions that are accurate in the total variation distance. This represents joint work with Jose Antonio Perez and Jonathan Goodman.

8. **James KUELBS**, University of Wisconsin at Madison, and **Joel ZINN**, Texas A&M University

   Some remarks on the CLT and the Compact LIL

   The classical limit theorems for i.i.d. data, namely the law of large numbers, the central limit theorem (CLT), and the law of the iterated logarithm (LIL), have counterparts when the observations are random vectors in a separable Banach space, but only the law of large numbers has exactly the same form as in the finite dimensional setting. In particular, necessary conditions for both the CLT and the LIL include not only conditions on the distribution of the data, but also on the partial sums of the data. What we seek to do here, is to try to understand the CLT and the LIL in terms of the data itself. Of course, some new ingredient must be introduced, and what we add is to allow slight modifications of the data. In particular, we restrict our modifications to be continuous, and to be no larger than a fixed small number, or in some cases a fixed small proportion of the magnitude of the data itself. What we find is that these modifications allow both the CLT and the LIL in a surprisingly broad setting, often where neither theorem held without additional assumptions.

   In mathematical terms we let B denote a separable Banach space with norm $\|\cdot\|$ and let μ be a probability measure on B for which linear functionals have mean zero and finite variance. One then has as a subspace (rarely closed) a Hilbert space (the closure, under the $L^2(\mu)$ norm, of the image of the covariance operator on B). For each $\epsilon > 0$ and $x \in B$, there is a unique point, $T_\epsilon(x)$, with minimum L^2 norm in the $\|\cdot\|$ ball of radius $\epsilon > 0$ and center x. If X is a random variable in B with law μ, under certain very weak conditions, we obtain Central Limit Theorems and Laws of the Iterated Logarithm for $T_\epsilon(X)$ and certain iterates of such a quantity, even when X itself fails to satisfy the corresponding limit theorem. The motivation for the use of the mapping $T_\epsilon(\cdot)$ comes from the large deviation rates for the Gaussian measure determined by the covariance of X, provided this Gaussian measure exists. However, this is only motivation, and our results apply even when this Gaussian law fails to exist.

9. **Peter NEY**, University of Wisconsin at Madison

   Large Deviations of Markov Chains (joint work with Alex de Acosta)

   Let $\{X_n : n = 1, 2, \ldots\}$ be an irreducible Markov chain taking values in a general state space S, $f : S \mapsto E$ a separable Banach space, μ an initial measure, $S_n = \sum_{i=1}^n f(X_i)$, $\mu_n(\cdot) = P_\mu(S_n/n \in \cdot)$. In Ann Probability (1998), we established an large deviation lower bound for $\{\mu_n(\cdot) : n = 1, 2, \ldots\}$, with a rate function Λ^* expressed in terms of the convergence parameter of the transform kernel of the Markov chain.

   This rate function can be shown to be tight (in the sense that it is also the rate for a corresponding large deviation upper bound) under some strong ergodicity conditions on $\{X_n\}$. In general it is not tight.

Using a "last visit to a small set" decomposition, we establish an large deviation upper bound rate $I(x) \leq \Lambda^*(x)$ (with equality for some $x \in E$). If $E = R^d$, and some further regularity conditions are satisfied, then one can show that the large deviation lower bound also holds with rate $I(\cdot)$, thus yielding a full large deviation principle with rate $I(\cdot)$.

10. **Steve SHREVE**, Carnegie Mellon University

    The Double Skorohod Map and Real-Time Queues (joint work with Lukasz Kruk, John Lehoczky, and Kavita Ramanan)

    Real-time queues are those in which customers have deadlines. Each arriving customer has a lead times, the time until the customer's deadline elapses, and the lead time runs down at unit rate per unit time. Each customer brings a certain amount of work for the server of the queue, and one can model the lead time/work profile of the queue by a measure on the real line that assigns mass equal to work at the position of the lead time. As the queue evolves, this so-called lead-time profile is a measure-valued process on R. As the queue enters heavy traffic, the limiting lead-time profile measure-valued process can be determined as a function of the limiting workload process for the queue.

    In this talk, we discuss real-time queues in which customers renege (leave the queue) when their deadlines elapse. We show in this case that the limiting workload process is a doubly reflected Brownian motion and we characterize the limiting lead-time profile process as a function of the limiting workload process. The analysis relies on a newly discovered formula for the operator that maps paths taking values in R into doubly-reflected paths taking values in $[0, a]$.

11. **Denis TALAY**, INRIA Sophia Antipolis, France

    Around Misspecified Models in Finance

    In this lecture I will present several results recently obtained jointly with various authors (C. Blanchet, R. Gibson, J. Jacod, A. Lejay, B. de Saporta, E. Tanre) on statistical and model risk issues in finance. I will focus on estimators of the Brownian dimension of an Ito process, and on the analysis of misspecified strategies derived from a stochastic control approach.

12. **R. J. WILLIAMS**, University of California at San Diego

    A Measure Valued Process in a Bandwidth Sharing Model with General Document Size Distributions

    We consider a stochastic flow level model of Internet congestion control introduced by Roberts and Massoulie. This model aims to capture connection level dynamics where bandwidth is shared dynamically and fairly amongst elastic document transfers. In contrast to a prior work by Kelly and Williams, the present paper allows interarrival times and document sizes to be generally distributed, rather than exponentially distributed. To describe the evolution of the system, measure valued processes are used to keep track of the residual document sizes of all flows in the network. We propose a fluid model (or formal functional law of large numbers approximation) for the flow level model operating under a weighted alpha-fair bandwidth sharing policy. Under mild conditions, the appropriately rescaled measure valued processes corresponding to a sequence of flow level models (with fixed network structure) are shown to be tight, and any weak limit point of the sequence is proved to be almost surely a fluid model solution. We characterize the invariant states for the fluid model and establish stability of the fluid model for two network topologies under a nominal condition. (This is based on joint work with H. C. Gromoll.)

13. **Walter WILLINGER**, AT&T Labs—Research

    On Internet Traffic as a Spatio-Temporal Process

    Traditionally, Internet traffic studies have focused on packet traces measured on a single link within an ISP's network. The statistical analysis and stochastic modeling of the resulting high-precision measurements have advanced mathematical research in the areas of self-similar stochastic processes, long-range dependence, and heavy-tailed distributions, which in turn has lead to deep new insights into the temporal aspects of Internet traffic over different time scales and at different layers in the protocol stack.

    However, an ISP's physical infrastructure consists of 100's or 1000's of such links. They are geographically dispersed and connected by routers or switches, and the Internet as a whole is made up of about 10,000 such ISPs. When viewed within this bigger context, Internet traffic becomes at once an even more challenging object to study, with potentially interesting and intriguing scale-dependent spatio-temporal features. I will discuss how the architectural design of the Internet gives rise to a natural, yet unconventional representation of network-wide traffic at different scales or levels of resolution and describe recent measurement efforts in support of a multi-scale analysis and modeling of the spatio-temporal behavior of Internet traffic.

14. **Thaleia ZARIPHOPOULOU**, University of Texas at Austin

    Dynamic Utilities

    In a general semimartingale asset model, we introduce a new class of utilities that process market information, and changes in risk attitude, forward in time. We solve the associated maximal expected utility problem and construct the optimal investment strategies and risk monitoring policies. Characterization results are also presented for utilities of affine-exponential type and stochastic risk tolerance.

2 Contributed Talks (including poster session)

1. **Marton BALAZS**, University of Wisconsin at Madison

   Construction of the Zero Range Process and a Deposition Model with Superlinear Growth Rates (joint work with Firas Rassoul-Agha, Timo Seppalainen and Sunder Sethuraman)

   The construction of nearest-neighbor interacting particle systems is well understood, due to methods initiated by T. M. Liggett and F. Spitzer, when the number of particles is locally (e.g. per site) bounded. The zero range process is a famous example of models with an unbounded number of particles per site. These processes are also constructed following the work of E. D. Andjel, when the jump rates are increasing and grow at most linearly fast with the local configuration. However, recent results involving shock-like measures shed light on processes with jump rates depending exponentially on the local configuration. The methods mentioned above do not seem to extend for constructing the dynamics of such a process. The talk will sketch a construction in the attractive case of (at most) exponentially growing jump rates. Estimates based on probabilistic coupling arguments are used to establish most of the usual Markov-semigroup machinery.

2. **Douglas BLOUNT**, Arizona State University

   Convergence, Tightness, and Separation Results for Probabilities on Skorokhod Path Space (joint work with Mike Kouritzin)

   Using topological arguments we simplify and generalize results of Ethier–Kurtz and Jakubowski. In some cases we eliminate separability and completeness assumptions while giving simpler and more transparent proofs.

3. **Vena Pearl BOÑGOLAN-WALSH, Jinqiao DUAN**, Illinois Institute of Technology, **Tamay ÖZGÖKMEN**, University of Miami

   Dynamics of Transport under Random Fluxes on the Boundary (Poster session)

   The impact of boundary noise on the dynamical evolution of the scalar transport equation in shear flows is studied, taking off from earlier studies in shear-flow dispersion in internal waves, a mechanism for horizontal mixing in the ocean. In particular, we model a gravity current evolving under an assumed shear-flow. The transport equation is deterministic, with a noise term at the inlet *boundary.* This was motivated by observed seasonal fluctuations in some known sources of salty, dense water in the oceans, like the Red Sea overflow, as well as observed thermal and saline anomalies in the thermohaline circulation.

   The noises used were: Wiener white, Wiener colored, Lévy white, and Lévy colored noise. Lévy processes form a more general class of processes which are generally non-Gaussian in distribution, and may have infinitely many jumps in finite time. They have been used to model pollutant point-sources, the flight time of particles in vortices, and linear and non-linear anomalous diffusion.

   The major finding was that white noises (Wiener and Lévy) and colored Wiener noise all have the effect of impeding the diffusion process, by as much as 33%. However, colored Lévy noise (non-Gaussian, time-correlated) does not have this effect on diffusion. This would suggest that time-correlation is more important in distinguishing noises than the distribution of the process that produced the noise. This also explains why Lévy colored noise showed great sensitivity to the stability parameter α, while Lévy white noise is unaffected by its stability parameter.

4. **Guillaume BONNET**, University of California at Santa Barbara

   Nonlinear SPDEs for Highway Traffic Flows (joint work with Gunnar Gunnarson)

   Mathematical models for the large scale dynamics of highway traffic flows, in the form of nonlinear PDEs, date back to the early work of Whitham. Since then, numerous improvements have been made on his basic model. Alternatively, microscopic stochastic models (cellular automaton) for car-by-car dynamics have been proposed. In this talk, I will present an SPDE model for highway traffic. This model can be thought as a "mezoscopic" scale description of the flow which incorporate the stochastic nature of real traffic as well as overall traffic dynamics. I will first give a brief derivation of this model and show that the corresponding equation is well posed. I will then present a numerical scheme and show its convergence. The calibration to real traffic data and some properties of this stochastic model will then be presented.

5. **C. CHEVALIER and F. DEBBASCH**, ERGA-LERMA, Université Paris 6, Paris, France

   Multi-Scale Diffusion on 2D Nearly Flat Riemannian Manifold (Poster session)

   Many biological phenomena involve diffusions on 2D nearly flat manifolds; metric fluctuations are however commonly neglected and these phenomena are then simply modeled by diffusions on the Euclidean plane. This poster presents a first attempt to evaluate the effect of metric fluctuations on diffusion phenomena. We suppose that the Fourier spectrum of the perturbation h around flat metric is bounded from below by a certain wave number $K^?$ and investigate only Brownian motion. The effective transport equation describing the Brownian motion in the nearly flat metric on scales much larger than $1/K^?$ is computed at order 2 in h. The correction terms to the usual (Euclidean plane) diffusion equation generically blow up exponentially with time. We thus reach the striking conclusion that, asymptotically, Brownian motion on a nearly flat 2D manifold cannot be approximated by standard planar Brownian motion, even on scales much larger than those over which the metric fluctuates.

6. **C. CHEVALIER and F. DEBBASCH**, ERGA-LERMA, Université Paris 6, Paris, France

   Diffusion Processes on Lorentzian Manifolds

   Constructing diffusion processes on Lorentzian manifolds has been a long standing problem. It has only been solved in 1997 when F. Debbasch, K. Mallick and J.P. Rivet extended the standard Ornstein–Uhlenbeck process to flat Lorentzian manifolds. Their construction has itself been extended recently to curved Lorentzian manifolds. We will review the basics of this extension and present as a physical application some new results pertaining to diffusions in expanding universes.

7. **Cristina COSTANTINI** Universita' di Chieti-Pescara
   Emmanuel GOBET ENSIMAG - INP Grenoble
   Nicole EL KAROUI Ecole Polytechnique, Paris

   Boundary Sensitivities for Diffusion Processes in Time Dependent Domains

   We study the sensitivity, with respect to a time dependent domain D_s, of expectations of functionals of a diffusion process stopped at the exit from D_s or normally reflected at the boundary of D_s. We establish a differentiability result and give an explicit expression for the gradient that allows the gradient to be computed by Monte Carlo methods. Applications to optimal stopping problems and pricing of American options, to singular stochastic control and others are discussed.

8. **Savas DAYANIK**, Princeton University

   Filling the Gap between American and Russian Options: Adjustable Regret

   We introduce a new exotic financial option with multiple exercise rights. This option spans naturally between the American and Russian options in terms of price and reduced regret.

We describe explicitly the value of the option, and an optimal exercise strategy whenever one exists. This is joint work with Michael Ludkovski.

9. **Tyrone E. DUNCAN and Yasong JIN**, UNIVERSITY OF KANSAS

Maximum Queue Length for a Queue with Gaussian Input

A fractional Brownian queueing model, that is, a fluid queue with a fractional Brownian motion as input, has been applied in network modeling since the self-similarity and long-range dependence were observed in Internet traffic. In this talk, we focus on a fluid queue with a general Gaussian input, which includes a fractional Brownian motion. The maximum queue length over an arbitrary time interval $[0, t]$ is studied. It is shown that a limit of the maximum queue length suitably normalized is determined by a suitable function of the asymptotic variance of the Gaussian input. Some queueing models, such as a queue with a heterogeneous fractional Brownian input and a queue with an integrated Gaussian input, will be discussed as examples. For a fractional Brownian model, these results are compared with some results in the literature.

10. **János ENGLÄNDER**, UNIVERSITY OF CALIFORNIA, SANTA BARBARA

The Compact Support Property for Measure-Valued Processes

The purpose of this talk is to give a rather thorough understanding of the compact support property for measure-valued processes corresponding to semi-linear equations of the form

$$u_t = Lu + \beta u - \alpha u^p \quad \text{in } \mathbb{R}^d \times (0, \infty), \;\; p \in (1, 2].$$

In particular, we were interested in how the interplay between the underlying motion (the diffusion process corresponding to L) and the branching affects the compact support property.

This joint work with R. Pinsky will appear in *Ann. Inst. H. Poincare* (B).

11. **Stewart ETHIER**, UNIVERSITY OF UTAH

Absorption-Time Distribution for an Asymmetric Random Walk

Consider the random walk on the set of nonnegative integers that takes two steps to the left (just one step from state 1) with probability $p \in (1/3, 1)$ and one step to the right with probability $1-p$. State 0 is absorbing and the initial state is a fixed positive integer j_0. Here we find the distribution of the absorption time. The absorption time is the duration of (or the number of coups in) the well-known Labouchere betting system. As a consequence of this, we obtain in the fair case ($p = 1/2$) the asymptotic behavior of the Labouchere bettor's conditional expected deficit after n coups, given that the system has not yet been completed.

12. **Jin FENG**, UNIVERSITY OF KANSAS AT LAWRENCE, AND UNIVERSITY OF MASSACHUSETTS AT AMHERST

A Uniqueness Result for Stochastic Scalar Conservation Law

Scalar conservation law is one nonlinear partial differential equation involving first order derivatives. In the early 70s, Kruzkov introduced a notion of weak solution called entropic solution, giving an existence and uniqueness theory for solution in L_1 space.

I will discuss a form of stochastic scalar conservation law where the randomness can be understood as entering the model through external source. Using Ito's calculus, a stochastic version of entropic solution (with second order derivative correction terms to the determinist version) is introduced. I will then outline how to obtain a type of contraction estimate and arrive at a uniqueness result. Existence results for such solution, however, is only partial at this point(i.e. only covering some special situations).

13. **Jerzy FILUS**, Oakton Community College
Lidia FILUS, Northeastern Illinois University

On Pseudonormal Markovian Processes

The class of pseudoaffine $R^n \mapsto R^n$, $(n \geq 2)$ transformations, being a new extension of the well known class of the ordinary affine mappings is introduced. The pseudoaffine transformations, when applied to n-variate random vectors of independent normal or exponential components, produce as outputs random vectors whose joint pdfs, called pseudonormal or pseudoexponential respectively, preserve some essential properties of the multivariate normal (or exponential) distributions. In particular, the multivariate normal pdfs are special cases of the pseudonormal. The classes of the new probability distributions are invariant with respect to the pseudoaffine transformations. The so obtained theory extends to infinite dimension as $n \to \infty$.

One then obtains an "infinite versions" of both affine and pseudoaffine as, say, $R^\infty \mapsto R^\infty$, transformations. These, when applied to any normal or exponential discrete time stochastic processes produce easily obtainable pseudonormal or pseudoexponential processes. Markovian subclasses as well as the martingales are singled out and investigated. As a next step the procedure of construction is extended to the stochastic pseudonormal processes with continuous time, where special emphasis is put on Markovian cases. The continuity in time is investigated. The corresponding pseudonormal generalization of the Wiener stochastic process and its potential applications are to be discussed.

14. **Jan HANNIG**, Colorado State University

Relative Frequencies of Generalized Simulated Annealing

We consider a class of non-homogeneous Markov chains arising in simulated annealing and related stochastic search algorithms. Using only elementary first principles, we analyze the convergence and rate of convergence of the relative frequencies of visits to states in the Markov chain. We describe in detail three examples, including the standard simulated annealing algorithm, to show how our framework applies to specific stochastic search algorithms—these examples have not previously been recognized to be sufficiently similar to share common analytical grounds. Our analysis, though elementary, provides the strongest sample-path convergence results to date for simulated annealing type Markov chains. Our results serve to illustrate that by taking a purely sample-path view, surprisingly strong statements can be made using only relatively elementary tools.

15. **Yasong JIN**, University of Kansas

Properties of Congestion Events for a Queue with Fractional Brownian Traffic
(joint work with Soshant Bali, Tyrone E. Duncan, Victor S. Frost) (Poster session)

In the early 1990s, researchers with Bellcore observed self-similarity and long-range dependence in LAN traffic. Since then many models have been proposed to study the impact of the phenomena on the queueing performance. A fractional Brownian model, proposed by Norros, is fundamental for analyzing the performance of a queue which has a self-similar and long-range dependent input. In this paper, a congestion event that occurs in a fractional Brownian model is defined. The properties of a congestion event, such as the sojourn time, the duration and the inter-congestion event time, are studied. A conditioned fractional Brownian motion is proposed to enable the analysis of a congestion event. Simulations were conducted for different traffic parameters. The analysis results are compared with the simulation results.

16. **Peter KACZMAREK, George RUS, Richard H. STOCKBRIDGE, Bruce WADE**, University of Wisconsin Milwaukee

    Numerical Solution of a Long-term Average Control Problem for Singular Stochastic Processes (Poster session)

    This paper numerically analyzes a long-term average stochastic control problem involving a controlled diffusion on a bounded region. The solution technique takes advantage of the linear programming formulation for the problem which relates the stationary measures to the generators of the diffusion. The restriction of the diffusion to the interval is accomplished through reflection at one end point and a jump operator acting singularly in time at the other end point. The numerical solutions exploit both a finite element method and a finite difference method for the estimation of the density of the stationary distribution and the determination of the optimal control.

17. **Yuriy V. KOLOMIETS**, Kent State University

    Fractional Stability for Functional CLT

    We consider the systems of random differential equations. The coefficients of the equations depend on a small parameter. The first equation, "slow" component, ordinary differential equation, has unbounded highly oscillating in space variable coefficients and random disturbances, which are described by the second equation, "fast" component, with periodic coefficients. The sufficient conditions for weak convergence as small parameter goes to zero of the solutions of the "slow" components to the certain random process are proved.

    Classical Diffusion Approximation Theorem (DAT) says, that drift coefficient of the approximated Stochastic Differential Equation (SDE) include a derivative with respect to a space variable of the unbounded coefficients (see, e.g. monograph of A. Skorokhod, and bibliography). So, we cannot apply classical DAT because of highly oscillating character of dependency on the small parameter of the unbounded coefficient of the "slow" component.

    On the other hand, we also cannot apply the Limit Theorem for SDE (in the sense of G. Kulinich, N. Portenko, M. Freidlin and A. D. Wentzell, S. Makhno) because the "slow" component is ODE and consequently has no nonzero diffusion coefficient (the presence of strongly positive diffusion coefficient is a necessary conditions for such kind of the theorems).

18. **Mike KOURITZIN**, University of Alberta; and MITACS; and Random Knowledge Inc., Canada

    On Designing and Applying Practical Particle Filters

The general area of particle filters has experienced great popularity and rapid development. Yet, most widely known filters are not well suited to certain applications in national security, network intrusion detection and targeted advertising.

In this talk, I will first motivate the use of particle filters from a mathematical perspective. Then, I will discuss some of the filter design constraints, related to speed, storage, adaptability and random environments, encountered in contract work done by the Prediction in Interacting Systems MITACS centre of excellence and by Random Knowledge Inc. Finally, I will contrast these constraints with the features of some of the more widely studied particle filters.

19. **Vladimir KURENOK** UNIVERSITY OF WISCONSIN AT GREEN BAY

On L_2-estimates of Stable Integrals with Drift

Let X be of the form $X_t = \int_0^t b_s dZ_s + \int_0^t a_s ds, t \geq 0$, where Z is a symmetric stable process of index $\alpha \in (1,2)$ with $Z_0 = 0$. We obtain various L_2-estimates for the process X. In particular, for $m \in N$, $t \geq 0$, and any measurable, nonnegative function f we derive the inequality $E \int_0^{t\wedge\tau_m(X)} |b_s|^\alpha f(X_s) ds \leq N \|f\|_{2,m}$. As an application of the obtained estimates, we prove the existence of solutions for the stochastic equation $dX_t = b(X_{t-})dZ_t + a(X_t)dt$ for any initial value $x_0 \in R$.

20. **Yoonjung LEE**, HARVARD UNIVERSITY

Modeling the Random Demand Curve for Stock: An Interacting Particle Representation Approach

The paper takes a microeconomic approach to modeling stock prices in illiquid markets. I propose a model in which the demand curve for stock is constructed from the private valuations of infinitely many interacting investors. Their valuation processes are assumed to be correlated through a common random factor and the stock price. The stock price is expressed as a quantile of the empirical measure of investors' valuations. With a particle representation of the system, I derive a stochastic partial differential equation for the empirical measure whose dynamics characterize the stock price dynamics. Adding a large trader to the pool of small investors, I examine the price impact of a large trade in the context of the Black–Scholes option pricing formula and the optimal trading policy of a large trader. Lastly, under a simple trading rule with transaction costs, the trading rate is quantified in terms of the bid and ask prices, the distribution of investors' valuations, and the interface between the distributions of owners' and non-owners' valuations.

21. **Chitro MAJUMDAR**, ETH-ZÜRICH, SWITZERLAND

Markov Continuous Time in Discrete Case: An Example

Consider the Markov modulated logarithmic Brownian motion

$$dX_t = B_t X_t dt + \sigma X_t dW_t$$

where B_t, W_t are independent and B_t is homogeneous Markov on $\{b_1, \ldots .b_k\}$ with intensity matrix Λ. We will proceed with writing down the Bellman equation for a Merton optimal investment/consumption problem in which B_t is not observable. The dynamic (differential programming) involving continuous time and a finite state space can be found in the recent literature, we will also try some approximation methods as is done usually by going from a discrete time frame to a continuous one through grids.

Furthermore in this Markov problem, we have looked at the investment problem when the risk free interest rate is in a switching regime.

22. **Ana MEDA**, Facultad de Ciencias, UNAM, México

Bounds for Dynamic Value at Risk

Let X_t be a stochastic process driven by a differential equation of the form $dX_t = \sigma(X_t)dW_t + b(X_t)dt, t > 0$ and let $X^*_{s,t} = \sup_{s \leq u \leq t} X_u$ be the maximum of the diffusion. We define several dynamic VaR type quantiles for this process and give bounds for both the VR quantile and the conditioned mean loss associated to it. We obtain upper bounds using the reflection principle and martingale techniques that can be applied to a large class of examples, as the Cox–Ingersoll–Ross model. Under certain conditions of boundedness on σ, and using only exponential martingales, we obtain upper and lower bounds for examples as the Vasicek model, and the Geometric Brownian Motion. We have been able extend the theory to allow for jumps driven by a compound Poisson process, prove our bounds are asymptotically optimal in some sense, and have an application for Ruin Probabilities.

This is joint work with Begoña Fernández, Laurent Denis, Daniel Hernández and Patricia Saavedra.

23. **N. Sri NAMACHCHIVAYA**, University of Illinois at Urbana-Champaign, and **Volker WIHSTUTZ**, University of North Carolina at Charlotte

Almost Sure Stability of Stochastic Delay Equations

We study the almost sure stability of delay differential equations with random parametric fluctuations which are modeled by a finite state Markov process. The study is motivated by stabilization/destabilization observed in numerical simulations of parametrically excited delay differential equations which arise, for instance, in pupil light reflection dynamics in physiology, logistic delay equations or population models in biology.

For small noise intensity we give an asymptotic expansion of the growth rate which determines the almost sure asymptotic stability of the stochastic system.

24. **Laurie SCOTT**, University of Missouri at Kansas City

Bayesian Inference via Filtering of Micro-movement Multivariate Stock Price Models with Discrete Noises (Poster session)

Presented here is a multivariate micro-movement model for asset prices. The model, as it is developed, reflects many of the stylized features of ultra-high frequency transaction (UHF), or trade-by-trade, data. In our model, we describe the trade-by-trade data as a collection of counting point process and then explicitly incorporate three types of 'noise' present in UHF data via a random transformation. We first study the likelihood, the marginal or integrated likelihood, the likelihood ratios, and the posterior and Bayes factors of the model and characterize them by evolution equations, such as filtering equations. We then develop the Bayesian inference including estimation and model selection via filtering for the model. Since those likelihoods, posteriors and the Bayes factor are continuous time and computationally unfeasible, we construct recursive algorithms to approximate them, and show the consistency of these algorithms. Empirical results for a simple model for two correlated stocks are provided.

25. **David SIRL**, University of Queensland, Australia

Bounds for the Decay Parameter of a Birth-Death Process

Determination of the decay parameter is critical to the analysis of absorbing continuous-time Markov chains, and there are several approximations and bounds for the particular case of the birth-death process. For example, Ingemar Nåsell (1996,1999,2001) derived approximations for the expected time to absorption starting with the quasi-stationary distribution (being the reciprocal of the decay parameter) for the well-known stochastic logistic or SIS epidemic model. This is a birth-death process on $\{0, 1, \dots, N\}$ with birth rates $\lambda_i = (\lambda/N)i(N-i)$ and death rates $\mu_i = \mu i$. Whilst Nåsell's approximations are generally good, their accuracy for given parameter values is unknown. We use a result of Mu-Fa Chen (2000) which gives upper and lower bounds that differ by less than a factor of four for a general absorbing birth-death process to approximate the decay parameter of the SIS epidemic model, as well as other examples of absorbing birth-death processes with both finite and infinite state spaces.

26. **Isaac M. SONIN**, University of North Carolina at Charlotte

A Decomposition-Separation Theorem for Markov Chains

Let M be a finite set, P be a stochastic matrix and $U = \{(Z_n)\}$ be the family of all finite Markov chains (MC) (Z_n) defined by M, P, and all possible initial distributions. The behavior of a MC (Z_n) is a classical result of Probability Theory derived in the 30's by A. Kolmogorov and W. Doeblin. If a stochastic matrix P is replaced by a sequence of stochastic matrices (P_n) and transitions at moment n are defined by P_n, then U becomes a family of *nonhomogeneous* MCs. There are numerous results concerning the behavior of such MCs given some specific properties of the sequence (P_n). But what if *there are no assumptions* about sequence (P_n)? Is it possible to say something about the behavior of the family U ? The surprising answer to this question is Yes. Such behavior is described by a theorem which we call a Decomposition-Separation (DS) Theorem, and which was initiated by a small paper of A. N. Kolmogorov (1936) and formulated and proved in a few stages in a series of papers including: D. Blackwell (1945), H. Cohn (1971, 1989) and I. Sonin (1987, 1991, 1996). The latter paper contains also a brief survey of related results including the problem of sufficiency of Markov strategies in some stochastic control problems. The DS Theorem also has a simple *deterministic interpretation* in terms of the behavior of the simplest model of an *irreversible process.* The irreversibility of this process manifests itself in the martingale property of some bounded random sequences defined by the family of MC U. Since the state space M is finite, $|M| = N$, these (sub)(super)martingales take no more that N values at each moment of time. Such martingales have extra properties which do not follow from Doob's well-known upcrossing lemma.

Some new results will be also presented but generally the DS Theorem leaves many open problems and leads probably to possible generalizations in other fields of mathematics besides probability theory.

27. **Richard H. STOCKBRIDGE**, University of Wisconsin Milwaukee

Singular Stochastic Control via Linear Programming

This talk focuses on the control of singular stochastic processes. The dynamics of the processes are formulated in terms of a martingale problem for both absolutely continuous and singular generators associated with the process. A brief indication of the equivalence of an infinite-dimensional linear program with the original singular control problem will be presented. The LP is stated in terms of two "occupation" measures, one with respect to regular time and the other with respect to "singular" time. The talk will then demonstrate that the LP approach can be implemented numerically with success. Two variations of the bounded follower problem studied by Beneš, Shepp and Witsenhausen will be presented.

28. **Jason SWANSON**, University of Wisconsin–Madison

Variations of the Solution to a Stochastic Heat Equation

We consider the solution to a stochastic heat equation. This solution is a random function of time and space. For a fixed point in space, the resulting random function of time, $F(t)$, has a nontrivial quartic variation. This process, therefore, has infinite quadratic variation and is not a semimartingale. It follows that the classical Ito calculus does not apply. Motivated by heuristic ideas about a possible new calculus for this process, we are led to study modifications of the quadratic variation. Namely, we modify each term in the sum of the squares of the increments so that it has mean zero. We then show that these sums, as functions of t, converge weakly to a Brownian motion, B, which is independent of F. As a corollary, we find that a certain sequence of Riemann sum approximations to $\int 2FdF$ converge in law to $F^2 - B$.

29. **Kathryn TEMPLE**, Central Washington University

Particle Representation of an Exit Measure

We use an adaptation of a particle representation of Kurtz and Rodrigues to construct an exit measure and therefore solutions of the nonlinear elliptic PDEs associated with a certain class of superdiffusions.

30. **Joe WATKINS**, University of Arizona at Tucson

The Wright–Fisher Diffusion Process and an Application to Queues and Bacterial Recombination

The title says it all.

31. **Weibiao WU**, University of Chicago

Another Look at Dependence (Poster session)

What is dependence? This is a fundamental problem in the study of random processes. Wu (2005, *Proc. Natl. Acad. Sci. USA*, Vol 40) interpreted random processes as physical systems and then introduced physical dependence coefficients that quantify the degree of dependence of outputs on inputs. Such dependence measures provide a new framework for the study of random processes and shed new light on a variety of problems including robust estimation of linear models with dependent errors, nonparametric inference of time series, representations of sample quantiles, bootstrap for time series, moderate deviation theory for stationary processes, spectral density estimation among others.

32. **Zhengxiao WU**, University of Wisconsin–Madison

A Filtering Approach to Abnormal Cluster Identification (Poster session)

A series of events $X_1, X_2, \ldots$ occur at times $\tau_1, \tau_2, \ldots$. Each event is either "normal" or "abnormal". We model the observations as a marked point process with a randomly initiated and growing cluster which represents the "abnormal" events. Our goal is to compute the conditional probability that an observed event is abnormal in real time. Employing filtering techniques, we derive versions of the Zakai and Kushner–Stratonovich equations in our setting. An algorithm for calculating the exact solution is given under a Markov condition. For the general case, a particle algorithm is proposed to give an approximate answer. The algorithm was illustrated on an earthquake dataset.

33. **Yong ZENG**, University of Missouri at Kansas City

Statistical Analysis of the Filtering Models with Marked Point Process Observations: Applications to Ultra-High Frequency Data

Ultra-high frequency (UHF) (or trade-by-trade) data are naturally modeled as marked point process (MPP). In this talk, we present two lines of the general filtering models with marked point process observations. Line One has Markov signals and Line Two has long-memory signals driven by fractional Brownian motion. We focus on Line One and show it subsumes existing important models. The statistical foundations and their related filtering equations are studied. If time allows, some computational issues will be discussed. Simulation examples and applications to real tick-by-tick financial data will be provided.