Stewart N. Ethier
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editors

Optimal Play

Mathematical Studies of Games
and Gambling

Institute for the Study of Gambling
and Commercial Gaming
To the Chevalier de Méré and his present-day counterparts—
the players who pose the questions that analyses such as
those contained herein can answer
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