

Stewart N. Ethier
William R. Eadington
editors

Optimal Play

**Mathematical Studies of Games
and Gambling**

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To the Chevalier de Méré and his present-day counterparts—
the players who pose the questions that analyses such as
those contained herein can answer

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