Math 5750/6880-3, Assignment 7, Feb. 26, 2016

Note: It is not enough just to give the answer (using the online game solver, for example). You must demonstrate that your answer is correct.

1. Player II chooses a number $j \in \{1, 2, \ldots, n\}$ and I tries to guess what it is. If he guesses correctly, he wins $1/j$ from II. If he guesses incorrectly, there is no payoff. Set up the matrix of this game and solve.

2. Solve the game with payoff matrix

$$\boldsymbol{A} = \begin{pmatrix} 5 & 4 & 6 & 9 \\ 9 & 7 & 5 & 3 \\ 4 & 5 & 9 & 6 \\ 7 & 9 & 3 & 5 \end{pmatrix}.$$

Hint: Try the formula that assumes nonsingular $\boldsymbol{A}$.

3. Recall the symmetric game "competing subsets" discussed in class. For the $N \times N$ case,

$$a_{ij} = \frac{i}{N} - \frac{j}{N} - \text{sign}(i - j)\frac{i}{N}\frac{j}{N}, \quad i, j = 1, 2, \ldots, N.$$

Solve the game for $N = 7$.

Hint: (a) It will suffice to analyze the game with a_{ij} replaced by $N^2 a_{ij}$. (It's easier with integer entries.)

(b) Two rows and columns are strictly dominated.

(c) The resulting 5×5 game has a mixed strategy solution with only three strategies active (you may take this for granted). That is, there is an optimal mixed strategy $\boldsymbol{p}$ with only three nonzero components.