Math 5750/6880-3, Assignment 6, Feb. 19, 2016

1. In general, a sure-fire test may be stated thus: For a given $m \times n$ game, conjectured optimal strategies $(p_1, \ldots, p_m)$ and $(q_1, \ldots, q_n)$ are indeed optimal if the minimum of I's average payoffs using $(p_1, \ldots, p_m)$ is equal to the maximum of II's average payoffs using $(q_1, \ldots, q_n)$. Show that for the game with 8×8 matrix

$$\boldsymbol{A} = \begin{pmatrix} 0 & -6 & -13 & -20 & -27 & -34 & -41 & -48 \\ 6 & 0 & -2 & -8 & -14 & -20 & -26 & -32 \\ 13 & 2 & 0 & 4 & -1 & -6 & -11 & -16 \\ 20 & 8 & -4 & 0 & 12 & 8 & 4 & 0 \\ 27 & 14 & 1 & -12 & 0 & 22 & 19 & 16 \\ 34 & 20 & 6 & -8 & -22 & 0 & 34 & 32 \\ 41 & 26 & 11 & -4 & -19 & -34 & 0 & 48 \\ 48 & 32 & 16 & 0 & -16 & -32 & -48 & 0 \end{pmatrix}$$

the mixed strategies

$$\boldsymbol{p} = \boldsymbol{q} = \frac{1}{117}(0, 0, 32, 60, 0, 16, 0, 9)^T$$

are optimal for I and II. What is the value? What about

$$\boldsymbol{p} = \boldsymbol{q} = \frac{1}{87}(0, 0, 32, 36, 0, 16, 0, 3)^T?$$

2. Consider the matrix game with

$$\boldsymbol{A} = \begin{pmatrix} 3 & -3 & 0 \\ 2 & 6 & 4 \\ 2 & 5 & 6 \end{pmatrix}.$$

(a) Find a mixed strategy of Player I that guarantees him the same payoff against any pure strategy of Player II.

(b) Find a mixed strategy of Player II that guarantees him the same payoff against any pure strategy of Player I.

(c) Find the value of the game, and show that the two strategies you found in (a) and (b) are optimal strategies of the two players.

3. Player II chooses a number $j \in \{1, 2, 3, 4\}$, and Player I tries to guess what number II has chosen. If he guesses correctly and the number was j, he wins $10 - j$ dollars from II. If he guesses incorrectly and the guess was j, he loses j dollars. Set up the matrix of this game and solve.

Hint: First it is easy to find a mixed strategy for I that guarantees I an expected gain of 0. Is there a mixed strategy for II that guarantees II an expected loss of 0?