

1. Find the characteristic function of the 3-person game in strategic form as follows (each player has two pure strategies):

$$I : 1 \quad \begin{array}{cc} III : 1 & III : 2 \\ II : 1 & \left(\begin{array}{cc} (2, 1, 5) & (3, 4, 2) \\ (1, 5, 3) & (2, 2, 4) \end{array} \right) \\ II : 2 & \end{array}$$

and

$$I : 2 \quad \begin{array}{cc} III : 1 & III : 2 \\ II : 1 & \left(\begin{array}{cc} (3, 3, 0) & (2, 3, 4) \\ (4, 5, 2) & (1, 6, 1) \end{array} \right) \\ II : 2 & \end{array}$$

Be sure to show all of your work.

2. Recall the example from class (also in the book): A certain *objet d'art* is worth a_i dollars to Player i for $i = 1, 2, 3$. We assume $a_1 < a_2 < a_3$, so Player 3 values the object most. But Player 1 owns this object so $v(\{1\}) = a_1$. Player 2 and 3 by themselves can do nothing, so $v(\{2\}) = 0$, $v(\{3\}) = 0$, and $v(\{2, 3\}) = 0$. If Players 1 and 2 come together, the joint worth is a_2 , so $v(\{1, 2\}) = a_2$. Similarly, $v(\{1, 3\}) = a_3$. If all three get together, the object is still only worth a_3 , so $v(N) = a_3$.

We found the core analytically. This exercise is to find the core graphically. Use the graphical method used in Example 1 on page IV-9 (i.e., barycentric coordinates).

3. In Nassau County government there are 6 districts (labeled 1, 2, 3, 4, 5, 6), with each district having an assigned number of votes approximately proportional to its size. The respective vote allotments are 31, 31, 28, 21, 2, 2. The total number of votes is 115. If a majority of at least 58 votes is required for passage of legislation, it is easy to see that the three smaller districts can never influence the outcome and the three larger districts are equally powerful, so the Shapley value is $(1/3, 1/3, 1/3, 0, 0, 0)$. This is unfair, so the law was changed to require at least 63 votes for passage of legislation.

(a) First, find all winning coalitions S such that 5 belongs to S but $S - \{5\}$ is losing. (District 5 has 2 votes.)

Next, find all winning coalitions S such that 4 belongs to S but $S - \{4\}$ is losing. (District 4 has 21 votes.)

(b) Determine the components of the Shapley value for districts 5 and 4 (with 2 and 21 votes, respectively).