Math 5750/6880-3, Assignment 10, Apr. 1, 2016

1. Let $((p^*, 1-p^*)^T, (q^*, 1-q^*)^T)$ be a Nash equilibrium for the bimatrix game with bimatrix

$$\begin{pmatrix} (a_1, a_2) & (b_1, b_2) \\ (d_1, d_2) & (c_1, c_2) \end{pmatrix}.$$

Suppose that row 2 strictly dominates row 1, that is, $d_1 > a_1$ and $c_1 > b_1$. Prove that $p^* = 0$, using the definition of Nash equilibrium.

2. Suppose that the bimatrix game with bimatrix

$$(\boldsymbol{A}_1, \boldsymbol{A}_2) = \begin{pmatrix} (a_1, a_2) & (b_1, b_2) \\ (d_1, d_2) & (c_1, c_2) \end{pmatrix}$$

is such that neither $\boldsymbol{A}_1$ nor $\boldsymbol{A}_2$ has a saddlepoint. Find a Nash equilibrium $((p^*, 1-p^*)^T, (q^*, 1-q^*)^T)$ (give formulas for p^* and q^*).

3. Consider the bimatrix game with bimatrix

$$(\boldsymbol{A}_1, \boldsymbol{A}_2) = \begin{pmatrix} (a_1, a_2) & (b_1, b_2) \\ (d_1, d_2) & (c_1, c_2) \end{pmatrix}.$$

If it is an NTU cooperative game, the feasible set is just the convex hull of the four points $(a_1, a_2), (b_1, b_2), (c_1, c_2), (d_1, d_2)$. If it is a noncooperative game, the feasible set is the set

$$\{(u, v) = ((p, 1-p)\boldsymbol{A}_1(q, 1-q)^T, (p, 1-p)\boldsymbol{A}_2(q, 1-q)^T) \in \mathbf{R}^2 : p, q \in [0, 1]\}.$$

This set is not necessarily convex. Graph it in the case of the Prisoners' Dilemma type game,

$$\begin{pmatrix} (1, 1) & (10, 0) \\ (0, 10) & (2, 2) \end{pmatrix}.$$

In particular, is the point $(5, 5)$ in the (noncooperative) feasible set?