Math 5750/6880-1: Game Theory Name:________________________
Final Exam
May 4, 2016

You have a choice of any five of the six problems. (If you do all 6, each will count 1/6, so there is no advantage.) You may use one sheet of notes that you have prepared. Cell phone/Internet use is prohibited. Calculators are allowed but results must be exact for full credit (i.e., no rounding).

Put a single X in the space corresponding to the one problem you skipped.
DO NOT mark more than one space.

1. ______ 2. ______ 3. ______ 4. ______ 5. ______ 6. ______

☐ I want all 6 problems counted.

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1. Suppose that at each turn a player may (1) remove one chip if it is a whole pile, or (2) remove two or more chips and, if desired, split the remaining chips into two piles.

(a) Find the Sprague-Grundy function $g(x)$ for $x = 0, 1, 2, \ldots, 8$.

(b) Find all optimal first moves in the game with three piles of sizes 5, 6, and 8.

2. Consider the two-person zero-sum game with payoff matrix

$$\boldsymbol{A} = \begin{pmatrix} 3 & -3 & -2 \\ 2 & 6 & 4 \\ 2 & 5 & 6 \end{pmatrix}. \qquad \text{(Note that} \quad \boldsymbol{A}^{-1} = \frac{1}{64}\begin{pmatrix} 16 & 8 & 0 \\ -4 & 22 & -16 \\ -2 & -21 & 24 \end{pmatrix}.\text{)}$$

Solve the game, that is, find the value and optimal strategies for I and II.

Hint: There are at least two methods to get I's optimal mixed strategy. (i) Find a mixed strategy for I that makes II indifferent about which pure strategy he uses, i.e., solve $(p_1, p_2, p_3)\boldsymbol{A} = (V, V, V)$ for p_1, p_2, p_3, V. (ii) Use the formula involving invertible $\boldsymbol{A}$.

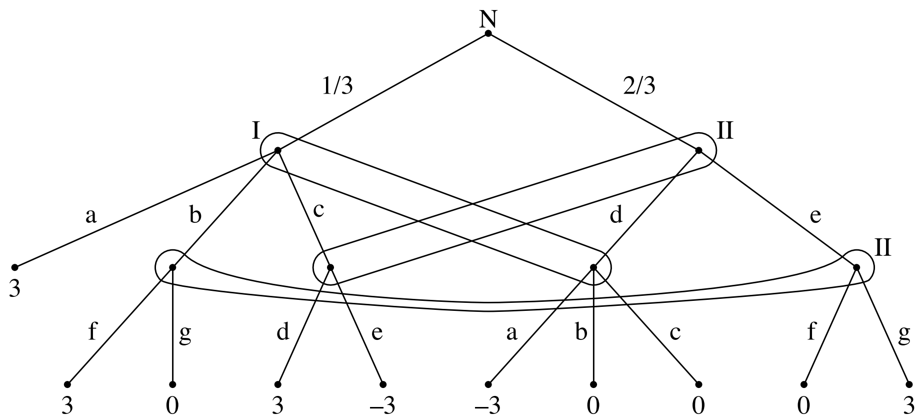

Figure 1: Game tree for Problem 3.

3. (a) Find the equivalent strategic form of the game with the game tree shown in Figure 1.

(b) Find optimal strategies for I and II and the value of the game. (Be very careful to get part (a) right before proceeding with part (b).)

4. In the noncooperative bimatrix game with bimatrix

$$\begin{pmatrix} (a_1, b_1) & (0,0) & (0,0) \\ (0,0) & (a_2, b_2) & (0,0) \\ (0,0) & (0,0) & (a_3, b_3) \end{pmatrix},$$

where $a_1, a_2, a_3, b_1, b_2, b_3$ are all > 0, find all Nash equilibria, at least one of which is not a pure Nash equilibrium.

5. Find the NTU-solution and the equilibrium exchange rate λ^* for the fixed threat point game with bimatrix

$$\begin{pmatrix} (1,0) & (-1,1) & (0,0) \\ (3,3) & (-2,9) & (2,7) \end{pmatrix}.$$

Hint: $\frac{1}{2} < \lambda^* < 4$.

6. The *gloves game* is a three-player game with characteristic function

$$v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\{1,2\}) = 0, \quad v(\{1,3\}) = v(\{2,3\}) = v(\{1,2,3\}) = 1.$$

Interpretation: Players 1 and 2 have a left-handed glove and Player 3 has a right-handed glove. The worth of a coalition is the number of complementary pairs of gloves it can form.

(a) Find the core.

(b) Find the Shapley value.