Math 5750-1: Game Theory Name:_____
Final Exam
May 5, 2015

You have a choice of any five of the six problems. (If you do all 6, each will count 1/6, so there is no advantage.) You may use one sheet of notes that you have prepared. Cell phone/Internet use is prohibited. Calculators are allowed but results must be exact for full credit (i.e., no rounding).

Put a single X in the space corresponding to the problem you OMITTED.

1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____

☐ I want all 6 problems counted.

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1. (a) Consider the subtraction game with subtraction set equal to the perfect squares (1, 4, 9, 16, ...). (In other words, a legal move consists of removing a perfect square number of chips from the pile. Last player to move wins.) Find the SG-function $g(x)$ for $x = 0, 1, 2, \ldots, 12$.

Hint: The ordinary sum of the 13 SG values should be 9. If it is not, you have made a mistake. Find your error(s) before proceeding.

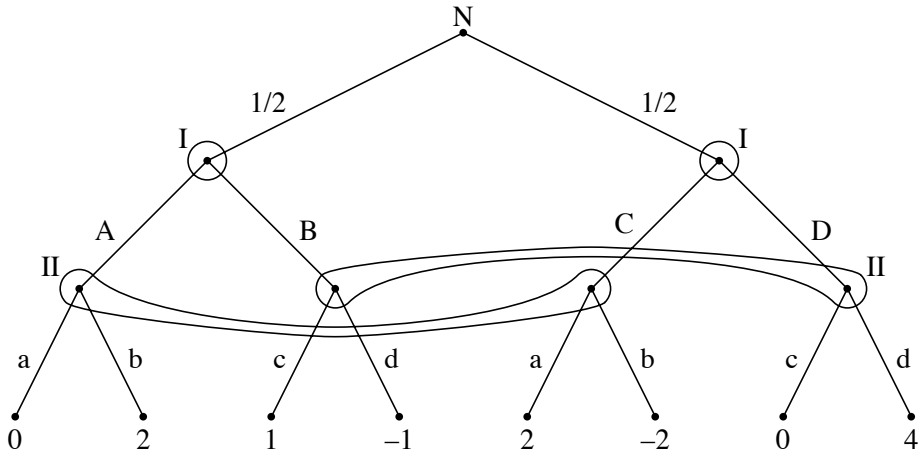
(b) Find *all* winning first moves, if any, for the three-pile version of the game with pile sizes 8, 9, 10.

2. Two players simultaneously choose positive integers in $\{1, 2, 3, \ldots, 12\}$. The goal to choose an integer larger but not too much larger than one's opponent chooses, but here "too much larger" is determined multiplicatively rather than additively. Solve the following example: The player whose number is larger but less than three times as large as the opponent's wins 1. But the player whose number is three times as large or larger loses 2. If the numbers are the same, there is no payoff.

(a) Note this is a symmetric game, and show that dominance reduces the game to a 3 by 3 matrix. (Do not just draw lines through rows and columns. Instead, say "row 2 dominates row 4" or something to that effect.)

(b) Solve the game by finding an optimal mixed strategy (valid for either player). We know the game's value is 0.

3. (a) Find the equivalent strategic form of the game with the game tree:



(b) Solve the game. (This part depends heavily on getting part (a) right. So first check your answer to part (a).)

4. Consider the non-cooperative bimatrix game:

$$(\boldsymbol{A}, \boldsymbol{B}) = \begin{pmatrix} (3,4) & (2,3) & (3,2) \\ (6,1) & (0,2) & (3,3) \\ (4,6) & (3,4) & (4,5) \end{pmatrix}.$$

(a) Find the safety levels, and the maxmin strategies for both players.
(b) Find as many Nash equilibria as you can. (Use strict dominance if possible.)

5. Find the NTU-solution and the equilibrium exchange rate λ^* for the following fixed threat point game.

$$(\boldsymbol{A}, \boldsymbol{B}) = \begin{pmatrix} (1,5) & (2,2) & (0,3) \\ (4,2) & (1,0) & (1,2) \\ (5,0) & (2,3) & (0,3) \end{pmatrix}.$$

6. (Two large political parties and three smaller ones.) Consider the weighted majority game with two large parties with 1/3 of the votes each and three smaller parties with 1/9 of the votes each. Find the characteristic function and the Shapley value.