Math 5040, Fall 2009
Assignment 1 on simulation
Due Sept. 23, 2009

Theoretical problems

1. (a) Simulating a standard normal is tricky, but consider the following idea. Let X and Y be independent $N(0,1)$ random variables. Let (R, Θ) be the polar coordinates corresponding to the point (X, Y). Then $R^2 = X^2 + Y^2$ has the chi-squared distribution with 2 degrees of freedom, which is exponential with mean 2, and Θ is uniform on $[0, 2\pi)$ and independent of R^2. (This can be proved from the transformation of densities formula—you need not provide the details.) Use these facts to describe an algorithm for simulating a standard normal random variable Z.

2. Suppose we want to simulate a gamma random variable (density $f(x) = \lambda^n x^{n-1} e^{-\lambda x}/(n-1)!$) using the rejection method and exponential distribution with density $g(y) = \beta e^{-\beta y}$ with $\beta < \lambda$, which we know how to simulate. Choose β to make your algorithm as efficient as possible (β may depend on λ and n), minimizing c of the rejection method. Then describe your algorithm as simply as possible.

Computer problems

3. Write a program in your favorite programming environment to simulate $E\big[|Z|\big]$, using the algorithm of problem 1. Choose your sample size to ensure accuracy to within 0.001 with confidence level 0.95. Write up your results so that they can be understood by someone who can't necessarily read your code.

4. Write a program to simulate the probability that a well-shuffled standard deck has at least one adjacent ace and jack, in either order. Choose your sample size to ensure accuracy to within 0.001 with confidence level 0.95. (If you are good at combinatorics, you may be able to find the exact answer. Even so, the problem is to simulate this probability.)

5. Let $(X_1, X_2, \ldots, X_{36})$ be multinomial$(900, 1/36, 1/36, \ldots, 1/36)$, and put $Y = \max_{1 \le i \le 36} X_i$. (To make this more concrete, think of a roulette wheel with no zeros, which is spun 900 times, and let Y be the frequency of the most frequent number.) Use a computer to simulate the distribution of Y, that is, simulate $P(Y = k)$ for $k = 0, 1, 2, \ldots, 60$, say. Run your program long enough to get reasonable accuracy (at least 3, maybe 4, decimal places).

6. Compare two methods for simulating a roll of a pair of dice. Method 1 simulates each die separately, then adds the results. Method 2 uses the distribution of the sum of two dice and the discrete inverse transformation method. Simulate 10 million dice rolls using each method, and time your programs. Which method is more efficient?