

Final Exam, Math 5010-2, Fall 2005, 12/14/05, Answers

1. (a) $\binom{16}{5}\binom{36}{0}$. (b) $4\binom{13}{2}\binom{13}{1}^3$. (c) $\binom{13}{5}\binom{4}{1}^5$.
2. (a) $1/2, 1/4$. (b) $1/52, 1/8, 1/104$. (c) Yes and No. (d) $17/104$.
3. $9/19, 6/19, 4/19$.
4. (a) $126/252, 70/252, 35/252, 15/252, 5/252, 1/252, 0, 0, 0, 0$. (b) $11/6$.
5. $f_X(x) = \pi^{-1}(1+x^2)^{-1}$, valid for all real x .
6. (a) $f_{X,Y}(x,y) = 1/(5x)$ if $1 \leq y \leq x \leq 5$, 0 otherwise.
 (b) $137/300, 77/300, 47/300, 27/300, 12/300$.
 (c) Given $Y = 1$, values are $60/137, 30/137, 20/137, 15/137, 12/137$.
 Given $Y = 2$, values are $0, 30/77, 20/77, 15/77, 12/77$.
 Given $Y = 3$, values are $0, 0, 20/47, 15/47, 12/47$.
 Given $Y = 4$, values are $0, 0, 0, 15/27, 12/27$.
 Given $Y = 5$, values are $0, 0, 0, 0, 1$.
 (d) No, since $Y \leq X$.
7. (a) With $p = 1/365$, mean is $365\binom{200}{2}p^2(1-p)^{198}$.
 (b) Variance is $365\binom{200}{2}p^2(1-p)^{198}\left[1 - \binom{200}{2}p^2(1-p)^{198}\right]$
 $+ (365)(364)\left[\binom{200}{2,2,196}p^2p^2(1-2p)^{196} - \left(\binom{200}{2}p^2(1-p)^{198}\right)^2\right]$.
8. (a) Chebyshev implies that this probability is at least $-1/3$, so it provides no useful information.
 (b) $2\Phi(\sqrt{3}/2) - 1$.