Final Exam, Math 5010–2, Fall 2005, 12/14/05   Name: _____

There are 8 problems, one from each chapter. You must do any 6.

1. Out of the 2,598,960 5-card poker hands, how many have ...
(a) ... only cards ranked J or higher (J,Q,K,A)?
(b) ... all 4 suits represented?
(c) ... 5 distinct denominations?
Note: There are no probabilities in this problem.

2. Two decks of cards, one with blue backs and one with green backs, are mixed together, for a total of 104 cards. A card is drawn at random. Let A be the event that it has the denomination of ace. Let B be the event that it has a blue back. Let C be the event that it has the suit of club. (a) Note that $P(A) = 8/104 = 1/13$. Find $P(B)$ and $P(C)$.
(b) Note that $P(AB) = 4/104 = 1/26$. Find $P(AC)$, $P(BC)$, and $P(ABC)$.
(c) Are A, B, C independent? Are they mutually exclusive?
(d) Find the probability of the event that at least 2 of the events A, B, and C occur. (Use a Venn diagram.)

3. There are 12 balls, of which 4 are white, in an urn. Three players—A, B, and C—successively draw from the urn, A first, then B, then C, then A, and so on. The winner is the first one to draw a white ball. Find the win probabilities for each player if each ball is replaced after it is drawn. Simplify your answers using $\sum_{n=1}^{\infty} a^n = a/(1-a)$ if $|a| < 1$.

4. Five men and five women are ranked according to their scores on an examination. Assume that no two scores are the same and all 10! possible rankings are equally likely. Let X denote the highest rank achieved by a woman (for instance, $X = 2$ if the top ranked person was male and the next ranked person was female).
(a) Find $P\{X = i\}$ for $i = 1, 2, 3, ..., 9, 10$.
(b) Give an expression for $E[X]$. You need not evaluate it, but make it clear that you could.

5. Let Q be uniformly distributed over the interval $(-\pi/2, \pi/2)$. Find the probability density function of the random variable $X = \tan Q$. Recall $(d/dq)\tan q = \sec^2 q$ and $(d/dx)\tan^{-1} x = 1/(1+x^2)$. For what values is your formula valid?

6. Choose a number at random from the set of numbers $\{1, 2, 3, 4, 5\}$. Now choose a number at random from the subset no larger than X, that is, from $\{1, ..., X\}$. Call this second number Y.
(a) Find the joint probability mass function of X and Y.
(b) Find the marginal probability mass function of Y.
(c) Find the conditional probability mass function of X given that $Y = i$. Do this for $i = 1, 2, 3, 4, 5$.
(d) Are X and Y independent? Justify your answer.

7. For a group of 200 people (none of whom is born on Feb. 29) compute:
(a) the expected number of days of the year that are birthdays of exactly 2 people in the group.
(b) the variance of the number of days of the year that are birthdays of exactly 2 people in the group.

8. 2500 random numbers are generated by a computer. These can be considered independent random variables, each with the uniform distribution on $[0, 1]$.
(a) What does Chebyshev's inequality say about the probability that their average will lie between 0.495 and 0.505?
(b) Approximate the probability in (a) using the central limit theorem. Express your answer in terms of the cumulative distribution function of the standard normal.

Just before turning in your exam, put an X in two of the boxes below to indicate omitted problems. Leave the other 6 blank.

1. _____   2. _____   3. _____   4. _____   5. _____   6. _____   7. _____   8. _____