

Math 3210-3

HW 9

Due Friday, September 21, 2007

NOTE: You are only required to turn in problems 1, 2, 5, and 6.

The Completeness Axiom

- ♣ For each of the following subsets of \mathbb{R} , describe the set of all upper bounds for the set. If the set is not bounded above you can write "NOT BOUNDED ABOVE." Also give the supremum and maximum of the set, if they exist.
 - the set of odd integers;
 - $\{1 - \frac{1}{n} : n \in \mathbb{N}\}$;
 - $\{r \in \mathbb{Q} : r^3 < 8\}$;
 - $\{\sin x : x \in \mathbb{R}\}$.
- Show that the set $A = \{x : x^2 < 1 - x\}$ is bounded above, and then find its least upper bound. (Make sure you prove that it is the least upper bound.)
- Let S be a nonempty bounded subset of \mathbb{R} and let $k \in \mathbb{R}$. Define $kS = \{ks : s \in S\}$. Prove the following:
 - If $k \geq 0$, then $\sup(kS) = k \cdot \sup S$ and $\inf(kS) = k \cdot \inf S$.
 - If $k < 0$, then $\sup(kS) = k \cdot \inf S$ and $\inf(kS) = k \cdot \sup S$.
- Let A be a non-empty subset of \mathbb{R} and x an element of \mathbb{R} . Then prove the following:
 - $\sup A \leq x$ iff $a \leq x$ for every $a \in A$.
 - $x < \sup A$ iff $x < a$ for some $a \in A$.
- Let A and B be non-empty subsets of \mathbb{R} . Then prove the following:
 - ♣ $\sup(A - B) = \sup(A) - \inf(B)$.
 - If $A \subseteq B$, then $\sup A \leq \sup B$ and $\inf B \leq \inf A$.
- ♣ Prove: If $y > 0$, then there exists a unique $n \in \mathbb{N}$ such that $n - 1 \leq y < n$.
- Let $x \in \mathbb{Q}$, $x \neq 0$, and let y be irrational. Prove that xy is irrational.