

Math 3210-3

HW 7

Due Friday, September 14, 2007

The Natural Numbers and Induction

1. ♣ Prove that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$ for all $n \in \mathbb{N}$.
2. ♣ Prove that $5^{2n} - 1$ is a multiple of 8 for all $n \in \mathbb{N}$.
3. ♣ Let a sequence $\{x_n\}$ of numbers be defined recursively by

$$x_1 = 0 \text{ and } x_{n+1} = \frac{x_n + 1}{2}.$$

Prove by induction that $x_n \leq x_{n+1}$ for all $n \in \mathbb{N}$. Would this conclusion change if we set $x_1 = 2$?

4. For each $n \in \mathbb{N}$, let P_n denote the assertion “ $n^2 + 5n + 1$ is an even integer.”
 - (a) Prove that P_{n+1} is true whenever P_n is true.
 - (b) For which n is P_n actually true? What is the moral of this exercise?
5. ♣ Use Theorem 24 to prove that $n^2 < 2^n$ for all $n \geq 5$.