Math 3210-3
HW 26
Due Tuesday, December 4, 2007

Power Series

1. Find the radius of convergence R and the interval of convergence C for each series:

   (a) $\sum \frac{n^2}{2^n} x^n$

   (b) $\sum \frac{(-4)^{-n}}{n} x^n$

   (c) $\sum (2^{-n})(x-5)^{2n}$

2. ♣ Find the radius of convergence for $\sum \frac{(3n)!}{(n!)^2} x^n$.

3. ♣ Suppose that the series $\sum a_n x^n$ has radius of convergence 2. Find the radius of convergence of each series, where k is a fixed positive integer.

   (a) $\sum a_n^k x^n$

   (b) $\sum a_n x^{kn}$

   (c) $\sum a_n x^{n^2}$

4. ♣ Prove that the series $\sum_{n=0}^{\infty} a_n x^n$ and $\sum_{n=0}^{\infty} n a_n x^n$ have the same radius of convergence (finite or infinite).

Pointwise and Uniform Convergence

5. ♣ Let $f_n(x) = \frac{x^n}{n}$ for $x \in [-1, 1]$. Find $f(x) = \lim f_n(x)$ and determine whether or not the convergence is uniform on $[-1, 1]$. Justify your answer.

6. Let $f_n(x) = \frac{x}{x+n}$ for $x \geq 0$.

   (a) Show that $f(x) = \lim f_n(x) = 0$ for all $x \geq 0$.

   (b) Show that if $t > 0$, the convergence is uniform on $[0, t]$.

   (c) Show that the convergence is not uniform on $[0, \infty)$.

7. ♣ If (f_n) and (g_n) converge uniformly on a set S, prove that $(f_n + g_n)$ converges uniformly on S.

8. Determine whether or not the given series of functions converges uniformly on the indicated set. Justify your answers.

   (a) $\sum n^{-x}$ for $x > \sqrt{2}$

   (b) $\sum \frac{x^2}{n^2}$ for $x \geq 5$