

**Math 3210-3**  
**HW 23**

Due Tuesday, November 20, 2007

## The Fundamental Theorem of Calculus

1. ♣ Calculate

(a)  $\lim_{x \rightarrow 0} \left( \frac{1}{x} \right) \int_0^x e^{t^2} dt$

(b)  $\lim_{h \rightarrow 0} \left( \frac{1}{h} \right) \int_3^{3+h} e^{t^2} dt$

2. ♣ Let  $f$  be defined as follows:  $f(t) = 0$  for  $t < 0$ ;  $f(t) = t$  for  $0 \leq t \leq 1$ ;  $f(t) = 4$  for  $t > 1$ .

(a) Determine the function  $F(x) = \int_0^x f(t) dt$ .

(b) Sketch  $F$ . Where is  $F$  continuous?

(c) Where is  $F$  differentiable? Calculate  $F'$  at the points of differentiability.

3. ♣ Let  $f : [0, 1] \rightarrow \mathbb{R}$  be a continuous function with continuous second derivative  $f''$ , and  $f(0) = f(1) =$

0. Prove that if  $\int_0^1 f(x)f''(x) dx = 0$ , then  $f \equiv 0$ . (Hint: Integration by parts.)