

Math 3210-1

HW 2

Due Tuesday, August 28, 2007

There are 10 points possible on this assignment plus 2 extra credit points. Each problem is worth one point except problem 3 and the extra credit problem which are worth two points each. The problems with ♣ are problems which we could present in our problem sessions.

Techniques of Proof I

- Write the contrapositive of each implication.
 - If all roses are red, then all violets are blue.
 - G is normal if G is not regular.
 - If K is closed and bounded, then K is compact.
- Write the converse of each implication in Exercise 1.
- Provide a counterexample for each statement.
 - For every real number x , if $x^2 > 4$, then $x > 2$.
 - For every positive integer n , $n^2 + n + 41$ is prime.
 - Every triangle is right triangle.
 - No integer greater than 100 is prime.
 - Every prime number is odd.
 - For every positive integer n , $3n$ is divisible by 6.
 - No rational number satisfies the equation $x^3 + (x - 1)^2 = x^2 + 1$.
 - No rational number satisfies the equation $x^4 + (1/x) - \sqrt{x + 1} = 0$.
- ♣ Let f be the function given by $f(x) = 3x - 5$. Use the contrapositive implication to prove: If $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.
- Use the contrapositive implication to prove: If n^2 is an even number, then n is an even number. (Hint: A number is odd iff it can be written as $2k + 1$ for some integer k .)

Techniques of Proof II

- Prove: There exists an integer n such that $n^2 + \frac{3n}{2} = 1$. Is this integer unique?
- ♣ Prove: If x is a real number, then $|x - 2| \leq 3$ implies that $-1 \leq x \leq 5$.
- ♣ Prove: If $\frac{x}{x - 1} \leq 2$, then $x < 1$ or $x \geq 2$.
- ♣ Prove or give a counterexample: For every positive integer n , $n^2 + 3n + 8$ is even.

The following problem is for **Extra Credit**. I typically do not give extra credit problems, but I really liked this problem, and I think students should at least think about it even if they decide not to complete the problem.

- Consider the following sentences:
 - The nucleus of a carbon atom consists of protons and neutrons.

(b) Jesus Christ rose from the dead and is alive today.

(c) Every differentiable function is continuous.

Each of these sentences has been affirmed by some people at some time as being "true." Write an essay on the nature of truth, comparing and contrasting its meaning in these (and possibly other) contexts. You might also want to consider some of the following questions: To what extent is truth absolute? To what extent can truth change with time? To what extent is truth based on opinion? To what extent are people free to accept as true anything they wish?