

# Math 3210-3

## HW 19

Due Tuesday, November 6, 2007

### The Mean Value Theorem

1. Prove that  $|\cos x - \cos y| \leq |x - y|$ .
2. ♣ Suppose that  $f$  is differentiable on  $\mathbb{R}$  and that  $f(0) = 0$ ,  $f(1) = 1$  and  $f(2) = 1$ .
  - (a) Show that  $f'(x) = \frac{1}{2}$  for some  $x \in (0, 2)$ .
  - (b) Show that  $f'(x) = \frac{1}{7}$  for some  $x \in (0, 2)$ .
3. ♣ Show that  $ex \leq e^x$  for all  $x \in \mathbb{R}$ .
4. ♣ Show that  $\sin x \leq x$  for all  $x \geq 0$ . *Hint:* Show that  $f(x) = x - \sin x$  is increasing on  $[0, \infty)$ .
5. ♣ Suppose that  $f$  and  $g$  are differentiable on  $\mathbb{R}$ ,  $f(0) = g(0)$ , and  $f'(x) \leq g'(x)$  for all  $x \in \mathbb{R}$ . Prove  $f(x) \leq g(x)$  for  $x \geq 0$ .
6. Show that  $f(x) = \ln x$  is uniformly continuous on  $[1, \infty)$ .