Math 3210-3
HW 18
Due Friday, November 2, 2007

The Derivative

1. For each of the following functions defined on $\mathbb{R}$, give the set of points at which it is *not* differentiable. Sketches will be helpful. No proofs are required.
   (a) $e^{|x|}$
   (b) $\sin|x|$
   (c) $|\sin x|$
   (d) $|x| + |x-1|$
   (e) $|x^2 - 1|$
   (f) $|x^3 - 8|$

2. Let $f(x) = x^2 \sin\left(\frac{1}{x}\right)$ for $x \neq 0$ and $f(0) = 0$.
   (a) Use Theorems 81 and 82 to show that f is differentiable at each $a \neq 0$ and calculate $f'(a)$. Use, without proof, the fact that $\sin x$ is differentiable and that $\cos x$ is its derivative.
   (b) ♣ Use the definition to show that f is differentiable at $x = 0$ and that $f'(0) = 0$.
   (c) Show that f' is not continuous at $x = 0$.

3. Let $f(x) = x \sin\left(\frac{1}{x}\right)$ for $x \neq 0$, and $f(0) = 0$.
   (a) Observe that f is continuous at $x = 0$.
   (b) ♣ Is f differentiable at $x = 0$? Justify your answer.

4. Let $f(x) = x^2$ for x rational and $f(x) = 0$ for x irrational.
   (a) Prove that f is continuous at $x = 0$.
   (b) Prove that f is discontinuous at all $x \neq 0$.
   (c) ♣ Prove that f is differentiable at $x = 0$. *Warning:* You cannot simply claim $f'(x) = 2x$.