Math 3210-3
HW 17
Due Tuesday, October 30, 2007

Uniform Continuity

1. Which of the following continuous functions are uniformly continuous on the specified set? Justify your answers. Use any theorems from this section that you wish.
   (a) $f(x) = x^{17} \sin x - e^x \cos 3x$ on $[0, \pi]$
   (b) $f(x) = x^3$ on $[0, 1]$
   (c) $f(x) = x^3$ on $(0, 1)$
   (d) $f(x) = x^3$ on $\mathbb{R}$
   (e) $f(x) = \frac{1}{x^3}$ on $(0, 1]$
2. (a) Prove that if f is uniformly continuous on a bounded set S, then f is a bounded function on S.
   (b) Use part (a) to prove that $\frac{1}{x^2}$ is not uniformly continuous on $(0, 1)$.
3. Let $g(x) = x^2 \sin\left(\frac{1}{x}\right)$ for $x \neq 0$ and $g(0) = 0$.
   (a) Prove that $g(x)$ is continuous on $\mathbb{R}$.
   (b) Why is g uniformly continuous on any bounded subset of $\mathbb{R}$?
   (c) Is g uniformly continuous on $\mathbb{R}$?