Math 3210-3

HW 15

Due Tuesday, October 23, 2007

Limits of Functions

1. Sketch the function $f(x) = \frac{x}{|x|}$. Determine, by inspection, the limits $\lim_{x\to\infty} f(x), \lim_{x\to 0^+} f(x)$, $\lim_{x\to 0^-} f(x)$, $\lim_{x\to -\infty} f(x)$, and $\lim_{x\to 0} f(x)$ when they exist. Also indicate when they do not exist.

2. Find the following limits and prove your answers.

   (a) ♣ $\lim\limits_{x\to 0} |x|$

   (b) $\lim\limits_{x\to 0} \dfrac{x^2}{|x|}$

3. Let $f(x) = \dfrac{\sin x}{|x|}$ for $x \neq 0$. Find $\lim\limits_{x\to 0^+} f(x)$ and $\lim\limits_{x\to 0^-} f(x)$. Does $\lim\limits_{x\to 0} f(x)$ exist?

4. Let f, g and h be functions from D into $\mathbb{R}$, and let c be an accumulation point of D. If $f(x) \leq g(x) \leq h(x)$ for all $x \in D$ with $x \neq c$, and if $\lim_{x\to c} f(x) = \lim_{x\to c} h(x) = L$, then prove that $\lim_{x\to c} g(x) = L$.

Continuous Functions

5. ♣ Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^2 - 3x + 5$. Use the definition of a continuous function to prove that f is continuous at 2.

6. ♣ Let $f : D \to \mathbb{R}$ and define $|f| : D \to \mathbb{R}$ by $|f|(x) = |f(x)|$. Suppose that f is continuous at $c \in D$. Prove that $|f|$ is continuous at c.

7. ♣ Suppose that f satisfies $f(x + y) = f(x) + f(y)$, and that f is continuous at 0. Prove that f is continuous at a for all a.