Math 3210-3
HW 12
Due Wednesday, October 3, 2007

NOTE: Only turn in problems 1(d), 2(a), 2(c), 3(c), 4(a), and 7.

Sequences

1. Write out the first seven terms of each sequence.
   (a) $a_n = n^2$
   (b) $b_n = \dfrac{(-1)^n}{n}$
   (c) $c_n = \cos \frac{n\pi}{3}$
   (d) $d_n = \dfrac{2n+1}{3n-1}$
2. Using only the definition of a limit of a sequence, prove the following.
   (a) For any real number k, $\lim_{n\to\infty} \left(\dfrac{k}{n}\right) = 0$.
   (b) ♣ For any real number $k > 0$, $\lim_{n\to\infty} \left(\dfrac{1}{n^k}\right) = 0$.
   (c) $\lim_{n\to\infty} \dfrac{3n+1}{n+2} = 3$.
   (d) ♣ $\lim_{n\to\infty} \dfrac{\sin n}{n} = 0$.
   (e) $\lim_{n\to\infty} \dfrac{n+2}{n^2-3} = 0$.
3. Using any of the Theorems 47-49 or the examples we worked in class from section 4.1, prove the following.
   (a) ♣ $\lim_{n\to\infty} \dfrac{1}{1+3n} = 0$.
   (b) $\lim_{n\to\infty} \dfrac{4n^2-7}{2n^3-5} = 0$
   (c) $\lim_{n\to\infty} \dfrac{6n^2+5}{2n^2-3n} = 3$.
   (d) ♣ $\lim_{n\to\infty} \dfrac{\sqrt{n}}{n+1} = 0$.
   (e) $\lim_{n\to\infty} \dfrac{n^2}{n!} = 0$.
   (f) If $|x| < 1$, then $\lim_{n\to\infty} x^n = 0$.
4. Show that each of the following sequences is divergent.
   (a) $a_n = 2n$.
   (b) ♣ $b_n = (-1)^n$.
   (c) $d_n = (-n)^2$.
5. Suppose that $\lim s_n = 0$. If (t_n) is a bounded sequence, prove that $\lim(s_n t_n) = 0$.
6. Prove or give a counterexample: If (s_n) converges to s, then $(|s_n|)$ converges to $|s|$.
7. Suppose that $(a_n), (b_n)$, and (c_n) are sequences such that $a_n \le b_n \le c_n$ for all $n \in \mathbb{N}$ and such that $\lim a_n = \lim c_n = b$. Prove that $\lim b_n = b$. (This is called the squeeze theorem.)