Math 3210-3
HW 10
Due Tuesday, September 25, 2007

NOTE: You are only required to turn in problems 1-5, and 8-9.

Topology of the Reals

1. Find the interior of each set.
   (a) $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$
   (b) $[0,3] \cup (3,5)$
   (c) ♣ $\{r \in \mathbb{Q} : 0 < r < \sqrt{2}\}$
   (d) $\{r \in \mathbb{Q} : r \geq \sqrt{2}\}$
   (e) $[0,2] \cap [2,4]$
2. ♣ Find the boundary of each set in exercise 1.
3. Classify each of the following sets as open, closed, neither, or both.
   (a) $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$
   (b) $\mathbb{N}$
   (c) $\mathbb{Q}$
   (d) ♣ $\cap_{n=1}^{\infty} \left(0, \frac{1}{n}\right)$
   (e) $\left\{x : |x-5| \leq \frac{1}{2}\right\}$
   (f) $\{x : x^2 > 0\}$
4. ♣ Find the closure of each set in exercise 3.
5. If A is open and B is closed, prove that $A \setminus B$ is open and $B \setminus A$ is closed.
6. Let S and T be subsets of $\mathbb{R}$. Prove the following:
   (a) $\mathrm{cl}(\mathrm{cl}\, S) = \mathrm{cl}\, S$
   (b) $\mathrm{cl}(S \cup T) = (\mathrm{cl}\, S) \cup (\mathrm{cl}\, T)$
   (c) $\mathrm{cl}(S \cap T) \subseteq (\mathrm{cl}\, S) \cap (\mathrm{cl}\, T)$
   (d) Find an example to show that equality need not hold in part (c).
7. For any set $S \subseteq \mathbb{R}$, let $\overline{S}$ denote the intersection of all the closed sets containing S.
   (a) Prove that $\overline{S}$ is a closed set.
   (b) Prove that $\overline{S}$ is the smallest closed set containing S. That is, show that $S \subseteq \overline{S}$, and if C is any closed set containing S, then $\overline{S} \subseteq C$.
   (c) Prove that $\overline{S} = \mathrm{cl}\, s$.
   (d) If S is bounded, prove that $\overline{S}$ is bounded.

Compact Sets

8. Show that each subset of $\mathbb{R}$ is not compact by describing an open cover for it that has no finite subcover.
   (a) $[1,3)$

(b) $\mathbb{N}$

(c) ♣ $\{\frac{1}{n} : n \in \mathbb{N}\}$

(d) $\{x \in \mathbb{Q} : 0 \leq x \leq 2\}$

9. ♣ Prove that the intersection of any collection of compact sets is compact.