SOLUTIONS

Math 1210-1 Test 3

1. (4 pts) Solve:

$$\int 4\cos\theta\, d\theta$$

$$\boxed{= 4\sin\theta + C}$$

2. (8 pts) Solve the following differential equation. Give the general solution and the solution satisfying that when $x = 0$, $y = 2$.

$$\frac{dy}{dx} = \frac{3x - 7x^2}{y^2}$$

$$y^2\, dy = (3x - 7x^2)\, dx$$

$$\int y^2 dy = \int 3x - 7x^2\, dx$$

$$\boxed{\frac{1}{3}y^3 = \frac{3}{2}x^2 - \frac{7}{3}x^3 + C}$$ GENERAL SOLUTION

$$\frac{1}{3}(2)^3 = C \Rightarrow C = \frac{8}{3}$$

$$\boxed{\frac{1}{3}y^3 = \frac{3}{2}x^2 - \frac{7}{3}x^3 + \frac{8}{3}}$$ PARTICULAR SOLUTION

3. (3 pts) Write the following in sigma notation:

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{50}$$

$$\boxed{= \sum_{i=1}^{50} \frac{1}{i}}$$

4. Find the following values: (you may leave the answer to (a) unsimplified)

(a) (6 pts) $\sum_{i=1}^{100}(2i^2 - 9i + 4)$

$$= 2\sum_{i=1}^{100} i^2 - 9\sum_{i=1}^{100} i + 4\sum_{i=1}^{100} 1$$

$$\boxed{= 2\left(\frac{100(100+1)(2(100)+1)}{6}\right) - 9\left(\frac{100(100+1)}{2}\right) + 4(100)}$$

(b) (4 pts) $\sum_{j=1}^{n}\left(\frac{1}{a_j} - \frac{1}{a_{j+1}}\right)$ $= \left(\frac{1}{a_1} - \cancel{\frac{1}{a_2}}\right) + \left(\cancel{\frac{1}{a_2}} - \cancel{\frac{1}{a_3}}\right) + \dots + \left(\cancel{\frac{1}{a_n}} - \frac{1}{a_{n+1}}\right)$

$$\boxed{= \frac{1}{a_1} - \frac{1}{a_{n+1}}}$$

5. (6 pts) Write the specific formula needed to estimate the area under the graph of $y = x^2 + 4$ between $x = 0$ and $x = 3$ with n subintervals and using the right hand endpoints of the subintervals. **DO NOT SIMPLIFY OR SOLVE!!**

$$R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$\Delta x = \frac{3-0}{n} = \frac{3}{n}$$

$$x_i = \frac{3i}{n}$$

$$R_n = \sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\left(\frac{3}{n}\right)$$

$$\boxed{= \frac{3}{n}\sum_{i=1}^{n} \left(\frac{3i}{n}\right)^2 + 4}$$

6. (4 pts) What is the Mean Value Theorem for Integrals? You may use a picture, but be sure to explain what the picture represents.

IF f IS A CONTINUOUS FUNCTION ON $[a,b]$

THERE EXISTS A $c \in (a,b)$ SUCH THAT

$$\int_a^b f(x)\,dx = f'(c)(b-a).$$

7. (4 pts) Find the derivative of $G(x)$ if $G(x) = \int_{x^5}^{5} t^2 + \cos t\, dt.$ $= -\int_5^{x^5} t^2 + \cos t\, dt$

$$\boxed{G'(x) = -\left[(x^5)^2 + \cos(x^5)\right](5x^4)}$$

8. (7 pts) Evaluate:

$$\int_0^{\pi/4} \frac{\sin t}{\cos^4 t}\, dt$$

$u = \cos t$

$du = -\sin t\, dt$

$$= \int_1^{\frac{\sqrt{2}}{2}} \frac{\sin t}{u^4} \frac{du}{-\sin t} = -\int u^{-4} du = \frac{1}{3} u^{-3} \Big|_1^{\sqrt{2}/2}$$

$$= \frac{1}{3}\left[\left(\frac{\sqrt{2}}{2}\right)^{-3} - 1\right] = \boxed{\frac{1}{3}\left[2^{3/2} - 1\right]}$$

9. (4 pts) If the velocity of a particle traveling along a coordinate axis is given by $v(t) = 3t + 13$, what is the displacement of the particle from $t = 2$ to $t = 9$ seconds?

$v(t) = 3t + 13$

$$S = \int_2^9 3t + 13\, dt = \frac{3}{2}t^2 + 13t \Big|_2^9$$

$$\boxed{= \frac{3}{2}(9)^2 + 13(9) - \left[\frac{3}{2}(2)^2 + 13(2)\right]}$$