Math 1210-1 Test 2

1. (12 pts) Find $f'(x)$ for the following: **(Do not simplify.)**

   (a) $f(x) = (x^2 - 5x + 9)^{1/2}$

   $f'(x) = \frac{1}{2}(x^2-5x+9)^{-1/2}(2x-5)$ ✓

   (b) $f(x) = \dfrac{\cos x}{x^3 - 6x}$

   $f'(x) = \dfrac{(x^3-6x)(-\sin x) - (\cos x)(3x^2-6)}{(x^3-6x)^2}$ ✓

   (c) $f(x) = 5x^2 \sin x$

   $f'(x) = 5x^2(\cos x) + \sin x(10x)$ ✓

2. (5 pts) Find $\dfrac{d^4y}{dx^4}$ if $y = x^7 - 3x^5 + 9x^3 - 27x + 14$.

   $\frac{dy}{dx} = 7x^6 - 15x^4 + 27x^2 - 27 \quad \frac{d^2y}{dx^2} = 42x^5 - 60x^3 + 54x \quad \frac{d^3y}{dx^3} = 210x^4 - 180x^2 + 54$

   $\dfrac{d^4y}{dx^4} = 840x^3 - 360x$ ✓

3. (6 pts) Define the following:

   (a) Derivative (formal definition please)

   We define the derivative of a function f as f'; also, $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ if the limit exists; also the derivative tells us the slope of the tangent line to the function f at any point c. ✓

   (b) Operator

   An operator is a rule or function whose input is a function and whose output is another function. ✓

4. (8 pts) The curve γ is given implicitly by $x^2 + 2xy^2 + 5y = x + 1$. Find the slope of the line ℓ which is tangent to γ at $(2, -1)$.

$$2x + \left[2x\left(2y\frac{dy}{dx}\right) + y^2(2)\right] + 5\frac{dy}{dx} = 1$$

$$\frac{dy}{dx}\left[4xy + 5\right] = 1 - 2x - 2y^2$$

$$\frac{dy}{dx} = \frac{1 - 2x - 2y^2}{4xy + 5}$$

$$\frac{dy}{dx}\Big|_{(2,-1)} \frac{1 - 2x - 2y^2}{4xy + 5} \Rightarrow \frac{dy}{dx}\Big|_{(2,-1)} \frac{1 - 2(2) - 2(-1)^2}{4(2)(-1) + 5} \Rightarrow \frac{1 - 4 - 2}{-8 + 5} \Rightarrow \frac{-5}{-3} = \boxed{\frac{5}{3}}$$ ✓

5. (5 pts) Suppose that a tumor in a person's body has a spherical shape and that treatment is causing the radius of the tumor to decrease at a rate of 2 millimeters per month. At what rate is the volume decreasing when the radius is 3 millimeter? (Recall that $V = \frac{4}{3}\pi r^3$.)

$\frac{dr}{dt} = 2\text{mm}$ $\quad r = 3\text{mm}$

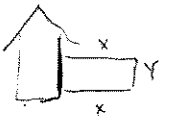
$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ mm/month

$$\frac{dV}{dt} = 4\pi(3)^2(-2) = 4\pi(9)(-2) = \boxed{-72\pi}$$

THE VOLUME IS DECREASING AT A RATE OF 72π mm^3/month. ✓

6. (10 pts) Farmer Bob wants to enclose part of his yard for his chickens. The enclosure will be rectangular, and one side of the yard will be against his barn. If he has 100 feet of fencing, what is the largest chicken pen he can enclose? (Draw a picture and label the sides.)



$2x + y = 100$ What is max area?

$y = 100 - 2x$

$A = xy \Rightarrow A = x(100 - 2x) \Rightarrow A = 100x - 2x^2$

$A' = 100 - 4x$

$0 = 100 - 4x$

$x = \frac{100}{4} \Rightarrow 25$

$A'' = -4 < 0$ max!

$2(25) + y = 100$

$50 + y = 100$

$y = 50$

$A = 25 \cdot 50 = 1250$ ✓

The largest fence is one w/ the x value of 25 ft. and a y value (the rest of the barn) is 50 ft long giving a max. area of 1250 ft²

25
50
1250

7. (4 pts) Bill traveled 160 miles in 2 hours and claimed he never exceeded 75 mph. Disprove Bill's claim.

$\frac{160}{2} = 80$

According to the Mean value Theorem Bill's Rate was 80 Mph at least once in order to travel 160 miles in 2 hrs. ✓

8. (10 pts) A function f has the following properties:

(a) f is a continuous functon

(b) $f(-1) = 0.5, f(0) = 0, f(1) = -1$

(c) $f'(-1) = f'(1) = 0$

(d) $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$

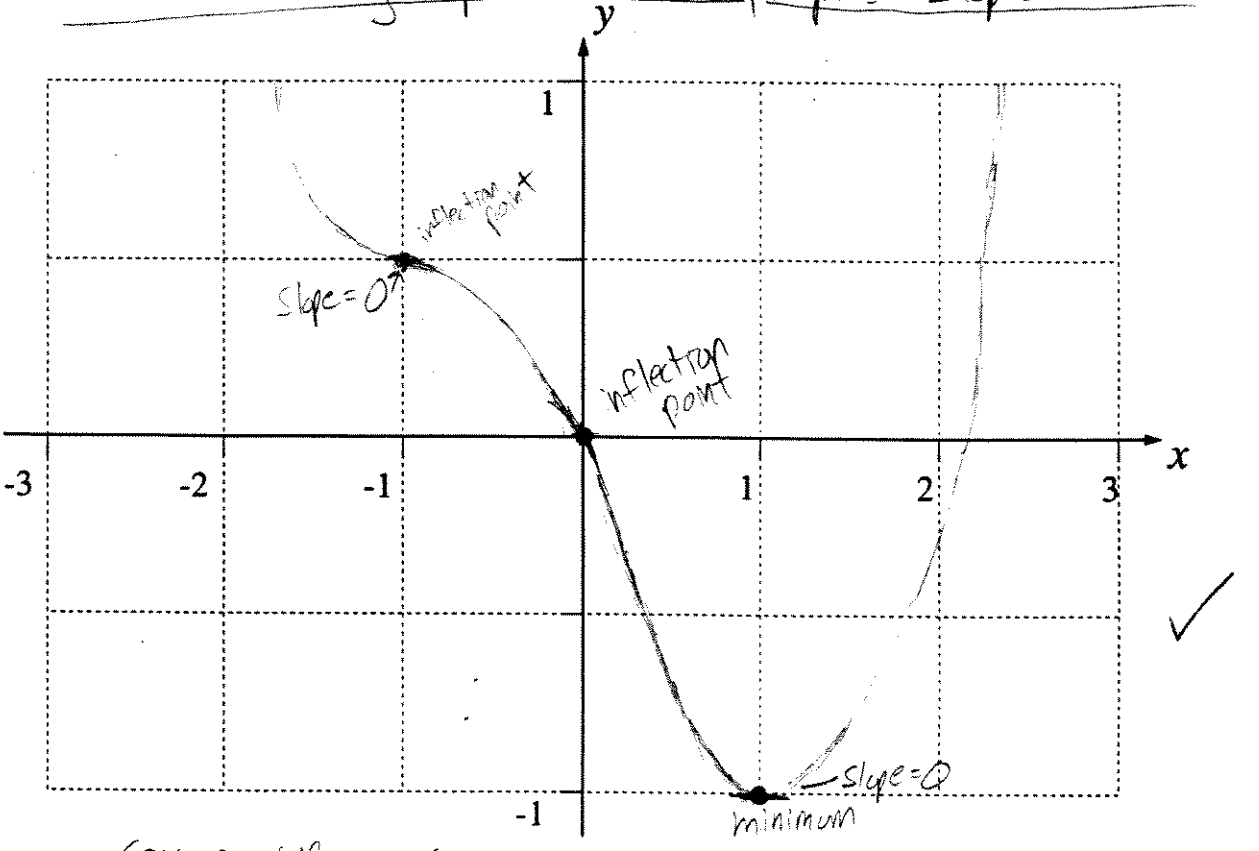
(e) $f''(-1) = f''(0) = 0$

(f) $f''(x) > 0$ for $x < -1$ and $x > 0$

(g) $f''(x) < 0$ for $-1 < x < 0$

Graph $y = f(x)$ on the window provided. Mark all interesting points on your graph and state what type of point they are.

neg slope | pos slope



concave up | concave down | concave up