SOLUTIONS

Math 1210-1 Test 1

1. (5 pts) Find the equation of the line passing through the point $(3, -1)$ and perpendicular to $7x - 9y = 3$.

$\perp$ To $7x - 9y = 3 \Rightarrow -9y = 3 - 7x \Rightarrow y = \frac{7}{9}x - \frac{1}{3}$

$\Rightarrow m = \frac{-9}{7}$

$$y - (-1) = \frac{-9}{7}(x - 3)$$

$$\boxed{y = \frac{-9}{7}(x-3) - 1}$$

2. (8 pts) If $f(x) = x^3 - 2x + 7$, find the indicated values. (**Do not simplify** b **and** c.)

(a) $f(-1) = (-1)^3 - (2)(-1) + 7 = -1 + 2 + 7 = \boxed{8}$

(b) $f(\frac{1}{t}) = \boxed{(\frac{1}{t})^3 - 2(\frac{1}{t}) + 7}$

(c) $f(x + h) = \boxed{(x+h)^3 - 2(x+h) + 7}$

3. (10 pts) Determine which of the following are functions. If not a function, **explain** why not.

(a)

x	y
♭	♠
♮	♡
♯	♣
♮	♢
♠	♠

NOT A FUNCTION BECAUSE THE SAME x-VALUE HAS 2 DIFFERENT y-VALUES

(b)

x	y
$\odot$	$\oplus$
$\star$	$\oslash$
$\ominus$	$\otimes$
$*$	$\diamond$
$\oslash$	$\oplus$

FUNCTION

(c)

x	y
$\odot$	†
♢	□
♠	♡
♡	♡
♢	△

NOT A FUNCTION BECAUSE $f(♢) = □$ AND △

(d)

x	y
♮	♭
♠	♡
♢	□
♮	♭
$\star$	♢

FUNCTION. ♮ IS MAPPED TO ONLY ♭.

4. (15 pts) Given $f(x) = 3x^2 - 5x + 8$ and $g(x) = \frac{x}{x-5}$, find each of the following: (**Do not simplify**.)

(a) $(f - g)(x)$

$$\boxed{= 3x^2 - 5x + 8 - \frac{x}{x-5}}$$

(b) $(f \cdot g)(x)$

$$= \boxed{(3x^2 - 5x + 8)\left(\frac{x}{x-5}\right)}$$

(c) $(f \circ g)(x)$

$$\boxed{= 3\left(\frac{x}{x-5}\right)^2 - 5\left(\frac{x}{x-5}\right) + 8}$$

(d) $(f \circ f)(x)$

$$\boxed{= 3(3x^2 - 5x + 8)^2 - 5(3x^2 - 5x + 8) + 8}$$

(e) $(g)^2(x)$

$$\boxed{= \left(\frac{x}{x-5}\right)^2}$$

5. (10 pts) What is the period, amplitude, vertical and horizontal shift of

$$y = 5\cos[2(x+3)] - 4?$$

PERIOD $= \frac{2\pi}{2} = \boxed{\pi}$

AMPLITUDE $= \boxed{5}$

VERTICAL SHIFT $= \boxed{\text{4 DOWN}}$

HORIZONTAL SHIFT $= \boxed{\text{3 LEFT}}$

6. (20 pts) Find the indicated limits.

(a) $\lim_{x \to 5} (x^2 - 7)$

$= 5^2 - 7 = 25 - 7 = \boxed{18}$

(b) $\lim_{x \to -3} \frac{x^2 - 4x - 21}{x + 3} = \lim_{x \to -3} \frac{(x+3)(x-7)}{x+3} = \lim_{x \to -3} x - 7 = -3 - 7 = \boxed{-10}$

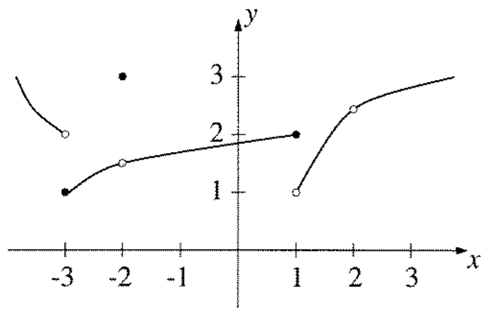
(c) $\lim_{t \to 0} \frac{\sin^2 t}{t} = \lim_{t \to 0} \sin t \left(\frac{\sin t}{t} \right) = \left[\lim_{t \to 0} \sin t \right] \left[\lim_{t \to 0} \frac{\sin t}{t} \right]$

$= 0(1) = \boxed{0}$

(d) $\lim_{x \to \infty} \frac{3x^4 + x^2}{7x^4 - 9x^3 + 7x - 1} \cdot \frac{\frac{1}{x^4}}{\frac{1}{x^4}} = \lim_{x \to \infty} \frac{3 + \frac{1}{x^2}}{7 - \frac{9}{x} + \frac{7}{x^3} - \frac{1}{x^4}} = \boxed{\frac{3}{7}}$

(e) $\lim_{t \to 2^-} \frac{x}{(x-2)(x-1)} = \lim_{x \to 0^-} \frac{2}{x(1)} = \boxed{-\infty}$

7. (22 pts) For the function f graphed in the figure below, find the following:



(a) $\lim_{x \to -2} f(x)$? $\boxed{1.5}$

(b) $f(-2)$. $\boxed{3}$

(c) Is f continuous at -2? Why or why not?

$\boxed{\text{No.}}$ BECAUSE $\lim_{x \to -2} f(x) \neq f(-2)$.

(d) $\lim_{x \to 1} f(x)$ $\boxed{\text{D.N.E.}}$

(e) $f(1)$ $\boxed{2}$

(f) Is f right continuous or left continuous or neither at 1?

$\boxed{\text{LEFT CONTINUOUS}}$

(g) $\lim_{x \to 2} f(x)$ $\boxed{2.5}$

(h) $f(2)$ $\boxed{\text{D.N.E.}}$

(i) Is f continuous at 2? Why or why not?

$\boxed{\text{No.}}$ $f(2)$ DOES NOT EXIST, SO $\lim_{x \to 2} f(x) \neq f(2)$.

(j) $\lim_{x \to -3^-} f(x)$ $\boxed{2}$

8. (10 pts) Show $h(x) = x^3 - x^2 + 7x - 2$ has a root between $x = 0$ and $x = 1$ using the Intermediate Value Theorem. (It might help to draw a picture.)

$h(0) = -2$

$h(1) = 1 - 1 + 7 - 2 = 5$

SINCE $h(0) < 0$ AND $h(1) > 0$, AND SINCE $h(x)$ IS A CONTINUOUS FUNCTION, ACCORDING TO THE MEAN VALUE THEOREM $\exists\, c \in (0,1)$ SUCH THAT $h(c) = 0$, i.e., h HAS A ROOT BETWEEN $x = 0$ AND $x = 1$.