CORRECTIONS
ON REVIEW 3

6. b) $\sum_{j=5}^{10} 2j - 17$

Review's Solutions

1. a) $\int 4x^3 - 2x^2 + 9\, dx = \boxed{x^4 - \frac{2}{3}x^3 + 9x + C}$

b) $\int x^{3/4}\, dx = \boxed{\frac{4}{7}x^{7/4} + C}$

c) $\int 2\sin x\, dx = \boxed{-2\cos x + C}$

d) $\int \cos^4 x \sin x\, dx$

$u = \cos x$

$du = -\sin x\, dx$

$\int u^4 \sin x \frac{du}{-\sin x} = -\int u^4 du = -\frac{1}{5}u^5 + C = \boxed{-\frac{1}{5}\cos^5 x + C}$

e) $\int (5x^2+1)\sqrt{5x^3+3x-2}\, dx$

$u = 5x^3+3x-2$

$du = (15x^2+3)dx$

$\int 5x^2+1\; u^{1/2} \frac{du}{3(5x^2+1)} = \frac{1}{3}\int u^{1/2} du = \frac{1}{3}\left(\frac{2}{3}u^{3/2}\right) + C = \boxed{\frac{2}{9}(5x^3+3x-2)^{3/2} + C}$

2. a) $\frac{dy}{dx} = -y^2 x(x^2+2)^4$

$-y^{-2} dy = x(x^2+2)^4 dx$

$u = x^2+2$

$du = 2x\, dx$

$-\int y^{-2} dy = \int x u^4 \frac{du}{2x}$

$y^{-1} = \frac{1}{5}u^5 + C = \frac{1}{5}(x^2+2)^5 + C$

$\boxed{y^{-1} = \frac{1}{5}(x^2+2)^5 + C}$

b) $y = 1,\ x = 0$

$1 = \frac{1}{5}(2)^5 + C$

$1 - \frac{32}{5} = C = -\frac{27}{5}$

$\boxed{y^{-1} = \frac{1}{5}(x^2+2)^5 - \frac{27}{5}}$

3. $a = -11\ ft/s^2$

$\frac{dv}{dt} = -11 \Rightarrow dv = -11\,dt = \int dv = \int -11\,dt \Rightarrow v = -11t + C_1$ AND AT $t = 0$

$v = 60\,mph = 88\,ft/s \Rightarrow 88 = C_1.\quad v = -11t + 88\quad ;\quad v = 0 = -11t + 88$

$\Rightarrow \frac{-88}{-11} = t = 8$ SEC CAR STOPS. $\frac{ds}{dt} = -11t + 88 \Rightarrow \int ds = \int -11t + 88\,dt$

$\Rightarrow S = -\frac{11}{2}t^2 + 88t + C_2$, + AT $t = 0$ $S = 0 \Rightarrow C_2 = 0$ SO CAR STOPS $S(8) = -\frac{11}{2}(8)^2 + 88(8)$ $\boxed{= 352\ ft}$

4. $V_0 = 55\,ft/s$

$\frac{dv}{dt} = -g \Rightarrow dv = -g\,dt \Rightarrow v = -gt + C_1$ AT $t = 0$, $v = 55 \Rightarrow C_1 = 55$

$\frac{ds}{dt} = v = -gt + 55 \Rightarrow ds = (-gt + 55)\,dt \Rightarrow S = \frac{-g}{2}t^2 + 55t + C_2$

AT $t = 0$ $S = 0 \Rightarrow 0 = C_2$

$S(t) = \frac{-g}{2}t^2 + 55t$. WANT $S(t)$ WHEN $v = 0 \Rightarrow$ MUST FIND

t FOR WHEN $v = 0$.

$v(t) = -gt + 55 \Rightarrow 0 = -gt + 55\quad gt = 55\quad t = \frac{55}{g}$

$$S\left(\frac{55}{g}\right) = \frac{-g}{2}\left(\frac{55}{g}\right)^2 + 55\left(\frac{55}{g}\right) = \frac{-55^2}{2g} + \frac{55^2}{g} = \boxed{\frac{55^2}{2g}\ ft}$$

5. a) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{100} = \boxed{\sum_{i=1}^{100} \frac{1}{i}}$

b) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{21} = \boxed{\sum_{i=1}^{11} \frac{(-1)^{i+1}}{2i-1}}$ (THERE ARE OTHER POSSIBILITIES)

c) $1 + 3 + 6 + 9 + 12 + \dots + 24 = \boxed{\sum_{j=1}^{8} 3j}$

d) $1 + 4 + 7 + 10 + 13 + \dots + 25 = \boxed{\sum_{j=1}^{8} 3j + 1}$

6. a) $\sum_{i=1}^{15} i^2 = \frac{15(15+1)(2\cdot 15+1)}{6} = \boxed{1240}$

b) $\sum_{j=5}^{10} 2j - 17 = 2(5) - 17 + 2(6) - 17 + 2(7) - 17 + 2(8) - 17 + 2(9) - 17 + 2(10) - 17$

$= 2(5+6+7+8+9+10) - 17(6) = 2(45) - 17(6) = \boxed{192}$

c) $\sum_{K=1}^{m+1} a_K - a_{K-1} = (a_1 - a_0) + (a_2 - a_1) + \ldots + (a_{m+1} - a_m) = \boxed{a_{m+1} - a_0}$

d) $\sum_{i=1}^{3^9} i^2 - 2i + 3 = \boxed{\frac{3^9(3^9+1)(3^9(2)+1)}{6} - 2\,\frac{3^9(3^9+1)}{2} + 3(3^9) =}$

7. a) ON THE 12^{TH} DAY, THE TRUE LOVE RECEIVES $\sum_{i=1}^{12} i$ GIFTS, ON THE 9^{TH} DAY THE TRUE LOVE RECEIVES $\sum_{i=1}^{9} i$ GIFTS, SO THE TOTAL # OF GIFTS GIVEN IN 12 DAYS IS

$$1 + \sum_{i=1}^{2} i + \sum_{i=1}^{3} i + \ldots + \sum_{i=1}^{12} i = \sum_{i=1}^{1} i + \sum_{i=1}^{2} i + \sum_{i=1}^{3} i + \ldots + \sum_{i=1}^{12} i = 1 + \frac{2(2+1)}{2} + \frac{3(3+1)}{2} + \ldots + \frac{12(12+1)}{2}$$

$\boxed{= 364 \text{ GIFTS}}$

b) # OF GIFTS GIVEN IN n DAYS IS $\boxed{\sum_{j=1}^{n} \sum_{i=1}^{j} i}$

8. $y = 2x + 2$ BETWEEN $x = 1$ & $x = 5$ n SUBINTERVALS, R.H. ENDPOINT

$\Delta x = \frac{5-2}{n} = \frac{3}{n}$

$x_i = 1 + \frac{3i}{n}$

$R_n = \sum_{i=1}^{n} f(x_i)\Delta x = \boxed{\sum_{i=1}^{n} \left[2\left(1 + \frac{3i}{n}\right) + 2\right] \frac{3}{n}}$

9. IF f IS A FUNCTION DEFINED ON $[a,b]$ AND IF $\lim_{\|P\| \to 0} \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$ EXISTS, THEN $\int_a^b f(x)dx = \lim_{\|P\| \to 0} \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$.

10. LET f BE CONTINUOUS ON $[a,b]$ & LET $x \in (a,b)$. THEN $\frac{d}{dx} \int_a^x f(t)dt = f(x)$.

11. LET f BE CONTINUOUS ON $[a,b]$ & LET F BE AN ANTIDERIVATIVE OF f ON $[a,b]$. THEN $\int_a^b f(x)dx = F(b) - F(a)$.

12. IF f IS CONTINUOUS ON $[a,b]$, $\exists\, c \in (a,b)$ s.t. $\int_a^b f(t)dt = f(c)(b-a)$.

13. $\int_0^2 (x^2-3)\,dx \qquad \Delta x = \frac{2-0}{n} = \frac{2}{n} \qquad x_i = \frac{2i}{n}$

$$R_n = \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^2 - 3\right]\left(\frac{2}{n}\right) = \sum\left(\frac{4i^2}{n^2} - 3\right)\frac{2}{n}$$

$$= \frac{2}{n}\left[\frac{4}{n^2}\left(\frac{n(n+1)(2n+1)}{6}\right) - 3n\right] = \frac{8}{n^3}\left(\frac{(n^2+n)(2n+1)}{6}\right) - 6$$

$$= \frac{8(2n^3+3n^2+n)}{6n^3} - 6 = \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2} - 6$$

$$\int_0^2 x^2-3\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2} - 6 = \frac{8}{3} - 6 = \boxed{\frac{-10}{3}}$$

CHECK: $\int_0^2 x^2-3\,dx = \frac{1}{3}x^3 - 3x \Big|_0^2 = \frac{1}{3}(2)^3 - 3(2) = \frac{8}{3} - 6$ ✓

14. a) $G(x) = \int_2^x 3t^3 - 8t + 2\,dt \Rightarrow \boxed{G'(x) = 3x^3 - 8x + 2}$

b) $F(x) = \int_1^{x^2} 5t + \cos t\,dt \Rightarrow$ ~~I~~ $u = x^2 \qquad \boxed{F'(x) = (5u + \cos u)\frac{du}{dx} = (5x^2 + \cos x^2)(2x)}$

c) $H(x) = \int_x^{\pi/2} \tan\theta\,d\theta = -\int_{\frac{\pi}{2}}^{x} \tan\theta\,d\theta \Rightarrow \boxed{H'(x) = -\tan x}$

15. a) $\int_1^4 \frac{s^4-8}{s^2}\,ds = \int_1^4 s^2 - 8s^{-2}\,ds = \frac{1}{3}s^3 + 8s^{-1}\Big|_1^4 = \left[\frac{1}{3}(4)^3 + 8\left(\frac{1}{4}\right)\right] - \left[\frac{1}{3}(1)^3 + 8(1)\right]$

$$= \frac{64}{3} + 2 - \left(\frac{1}{3} + 8\right) = \frac{64 + 6 - 1 - 24}{3} = \boxed{15}$$

b) $\int_4^{17} \sqrt{w}\,dw = \frac{2}{3}w^{3/2}\Big|_4^{17} = \boxed{\frac{2}{3}\left(17^{3/2} - 4^{3/2}\right)}$

$$\begin{array}{r} 49 \\ \times\ 9 \\ \hline 441 \end{array}$$

c) $\int_1^x t^5\,dt = \frac{1}{6}t^6\Big|_1^x = \boxed{\frac{1}{6}x^6 - \frac{1}{6}}$

d) $\int x^6(7x^7+\pi)^8 \sin\left[(7x^7+\pi)^9\right]dx \qquad u = (7x^7+\pi)^9$

$du = 9(7x^7+\pi)^8(49x^6)\,dx$

$$= \int x^6(7x^7+\pi)^8 \sin u \frac{du}{441(7x^7+\pi)^8 x^6} = \frac{1}{441}\int \sin u\,du = \frac{-1}{441}\cos u + C$$

$$\boxed{= \frac{-1}{441}\cos\left[(7x^7+\pi)^9\right] + C}$$

e) $\int_1^4 \frac{1}{\sqrt{t}\,(\sqrt{t}+1)^3}\,dt$ $\qquad u = t^{1/2}+1$

$\qquad du = \frac{1}{2} t^{-1/2}\,dt$

$$\int \frac{1}{t^{1/2}\,u^3}\,\frac{du}{\frac{1}{2}\cdot\frac{1}{\sqrt{t}}} = 2\int u^{-3}\,du = -u^{-2}\Big|_{t=1}^{t=4} = -(t^{1/2}+1)^{-2}\Big|_{t=1}^{t=4}$$

$$= -(4^{1/2}+1)^{-2} - \left[-(1^{1/2}+1)^{-2}\right] = \frac{-1}{9} + \frac{1}{4} = \frac{-4+9}{36} = \boxed{\frac{5}{36}}$$