Review 2 Solutions

1. a) $f(x) = 2\cos^3 x$

$$\boxed{f'(x) = 6\cos^2 x(-\sin x)}$$

b) $f(x) = x^2 \sin x$

$$\boxed{f'(x) = 2x\sin x + x^2 \cos x}$$

c) $f(x) = x^{2/3}$

$$\boxed{f'(x) = \frac{2}{3} x^{-1/3}}$$

d) $f(x) = \sqrt{\sin^2 x + 1} = (\sin^2 x + 1)^{1/2}$

$$\boxed{f'(x) = \frac{1}{2}(\sin^2 x + 1)^{-1/2}(2\sin x)(\cos x)}$$

e) $f(x) = \dfrac{x^3 + 5x^2 - 2x + 7}{x^2 - 3x + 9}$

$$\boxed{f'(x) = \frac{(3x^2 + 10x - 2)(x^2 - 3x + 9) - (x^3 + 5x^2 - 2x + 7)(2x - 3)}{(x^2 - 3x + 9)^2}}$$

2. Find $\dfrac{d^5 y}{dx^5}$ if $y = x^6 + x^2 + 1$

$$\frac{dy}{dx} = 6x^5 + 2x \qquad \frac{d^3y}{dx^3} = 120x^3$$

$$\frac{d^2y}{dx^2} = 30x^4 + 2 \qquad \frac{d^4y}{dx^4} = 360x^2$$

$$\boxed{\frac{d^5y}{dx^5} = 720x}$$

3. a) A <u>Function</u> is a rule which assigns to each x in the domain a single $f(x)$ in the range.

b) (Formal) We say $\lim_{x\to c} f(x) = L$ if for all $\varepsilon > 0$ there exists a $\delta > 0$ such that if $|x - c| < \delta$ that implies $|f(x) - L| < \varepsilon$.

(Informal) We say $\lim_{x\to c} f(x) = L$ if L is the obvious destination for the function as $x \to c$, i.e. if x gets close to c, then $f(x)$ gets close to L

c) If f is a function, we define the <u>derivative</u> of f, f', as

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$ if the limit exists.

d) An <u>Operator</u> is a rule or a function whose input is a function and whose output is another function.

e) AN ODD FUNCTION IS A FUNCTION WHICH IS SYMMETRIC TO THE ORIGIN, i.e. $f(-x) = -f(x)$.

f) AN EVEN FUNCTION IS A FUNCTION WHICH IS SYMMETRIC TO THE y-AXIS, i.e. $f(-x) = f(x)$.

4. $f(x) = x^5 - x^2 - 3x + 1$

$f(0) = 1$ AND $f(1) = -2$. SINCE f IS A POLYNOMIAL FUNCTION, AND POLYNOMIAL FUNCTIONS ARE CONTINUOUS, f IS A CONTINUOUS FUNCTION. ALSO, SINCE $-2 < 0 < 1$, BY THE INTERMEDIATE VALUE THEOREM, $\exists c$ BETWEEN 0 AND 1 SUCH THAT $f(c) = 0$, i.e. f HAS A ROOT BETWEEN 0 AND 1.

5. γ IS $x^4 + 3xy^3 + x^2y^2 = 5$

$$4x^3 + 3y^3 + 9xy^2\frac{dy}{dx} + 2xy^2 + 2x^2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(9xy^2 + 2x^2y\right) = -4x^3 - 3y^3 - 2xy^2$$

$$\frac{dy}{dx} = \frac{-4x^3 - 3y^3 - 2xy^2}{9xy^2 + 2x^2y}$$

$$\left.\frac{dy}{dx}\right|_{(-1,1)} = \frac{-4(-1)^3 - 3(1)^3 - 2(-1)(1)^2}{9(-1)(1)^2 + 2(-1)^2(1)} = \frac{4 - 3 + 2}{-9 + 2} = \frac{3}{-7} = \frac{-3}{7}$$

SLOPE OF THE TANGENT LINE TO γ AT $(-1,1)$ IS $\boxed{\frac{-3}{7}}$

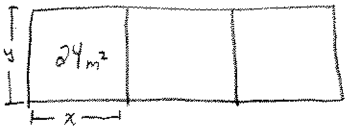
6. RADIUS GROWING AT A RATE OF 5 cm/s HOW FAST IS VOLUME INCREASING WHEN R = 15 cm?

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\left.\frac{dV}{dt}\right|_{r=15} = 4\pi(15)^2(5) = \boxed{4500\pi \text{ cm}^3/\text{s}}$$

7.



$f = 6x + 4y$

$+ \; xy = 24$

$y = 24x^{-1}$

$f(x) = 6x + 96x^{-1}$

$f'(x) = 6 - 96x^{-2}$

$0 = 6 - 96x^{-2}$

$6 = \frac{96}{x^2}$

$x^2 = \frac{96}{6}$

$x = \pm\sqrt{\frac{96}{6}}$

$\boxed{x = 4 \quad \Rightarrow \quad y = \frac{24}{4} = 6}$

$f''(x) = 192x^{-3}$

$f''(4) > 0 \Rightarrow$ MIN AT $x = 4$

8. 112 MILES IN 2 HRS $\Rightarrow$ AVERAGE SPEED WAS $\frac{112}{2} = 56$ mph.

BY THE MEAN VALUE THRM, AT SOME TIME WITHIN THOSE TWO HOURS, JUSTIN WAS TRAVELING AT 56mph. THEREFORE JUSTIN'S CLAIM IS WRONG.

9. CRITICAL POINTS: c IS A CRITICAL POINT OF A FUNCTION f IF c SATISFIES ANY OF THE FOLLOWING:

i) ~~x~~ c IS AN ENDPOINT OF A GIVEN INTERVAL

ii) $f'(c) = 0$

iii) $f'(c)$ IS UNDEFINED

EXTREME VALUES: b IS AN EXTREME VALUE OF A FUNCTION f IF EITHER OF THE TWO PROPERTIES BELOW HOLD:

i) $f(c) = b$ FOR c A CRITICAL POINT OF f AND $f(x) \leq b \;\; \forall x \in$ DOMAIN

ii) $f(c) = b$ FOR c A CRITICAL POINT OF f AND $f(x) \geq b \;\; \forall x \in$ DOMAIN.

INFLECTION POINTS: c IS AN INFLECTION POINT OF A FUNCTION f IF $f''(c) = 0$ AND f CHANGES FROM CONCAVE UP TO CONCAVE DOWN (OR VICE VERSA) AT c. INFLECTION POINTS OF A FUNCTION f CORRESPOND TO MAX/MIN VALUES OF $f'(x)$.

SLOPE: WE CAN DETERMINE THE SLOPE OF THE TANGENT LINE OF A FUNCTION f AT ANY POINT c IN ITS DOMAIN BECAUSE THE SLOPE $= f'(c)$. WE ALSO KNOW f IS INCREASING OR HAS POSITIVE SLOPE WHEN $f'(x) > 0$ AND IS DECREASING WHEN $f'(x) < 0$.

CONCAVITY: THE CONCAVITY OF A GRAPH OR FUNCTION DESCRIBES HOW THE SLOPE CHANGES. A FUNCTION IS CONCAVE UP WHEN $f''(x) > 0$ AND CONCAVE DOWN WHEN $f''(x) < 0$

10.