# HW 9

1. $f = 4y + 6x \quad \& \quad xy = 16\,m^2$

$y = \frac{16}{x}$

$f = 4\left(\frac{16}{x}\right) + 6x$

$f = 64x^{-1} + 6x$

$f' = -64x^{-2} + 6 = \frac{6x^2 - 64}{x^2}$

$0 = 6x^2 - 64$

$6x^2 = 64$

$x^2 = \frac{64}{6}$

$x = \pm\sqrt{\frac{64}{6}} = \sqrt{\frac{32}{3}}$

$f''(x) = 64(2)x^{-3}$

$f''\left(\sqrt{\frac{32}{3}}\right) > 0 \Rightarrow$ MIN AT $x = \sqrt{\frac{32}{3}}$

$\therefore$ $x = \sqrt{\frac{32}{3}}$ m, $y = 16\sqrt{\frac{3}{32}}$ m GIVES A MINIMUM FOR THE AMOUNT OF FENCE NEEDED

2. $C = 10\left[2y + 6x\right] + (2y)5$

$C = 20y + 60x + 10y$

$y = \frac{16}{x}$

$C = 30y + 60x$

$C = 30\frac{16}{x} + 60x$

$C = 480x^{-1} + 60x$

$C' = -480x^{-2} + 60 = \frac{60x^2 - 480}{x^2}$

$0 = 60x^2 - 480$

$480 = 60x^2$

$8 = x^2 \qquad x = 2\sqrt{2}$ m

$C'' = 480(2)x^{-3}$

$C''(2\sqrt{2}) > 0 \Rightarrow$ MIN AT $x = 2\sqrt{2}$

SO LEAST EXPENSIVE WHEN

$\boxed{x = 2\sqrt{2}\text{ m}}$ $\quad y = \frac{16}{2\sqrt{2}}$ m $\boxed{= 4\sqrt{2}}$

3. $v = a(R-r)r^2$

$v = aRr^2 - ar^3$

$v' = 2aRr - 3ar^2$

$0 = ar(2R-3r)$

$0 = r \qquad 3r = 2R$

$r = \frac{2}{3}R$

$v'' = 2aR - 6ar$

$v''(0) > 0 \Rightarrow 0$ GIVES MINIMUM

$v''(\frac{2}{3}R) = 2aR - 6a(\frac{2}{3}R) = 2aR - 4aR = -2aR < 0 \Rightarrow \boxed{r = \frac{2}{3}R \text{ GIVES MAX}}$

4. a) $T = \left(\frac{C}{2} - \frac{D}{3}\right)D^2 = \frac{C}{2}D^2 - \frac{1}{3}D^3$

$T' = CD - D^2 = D(C-D)$

$0 = D(C-D)$

$D = 0, \quad D = C$

$T'' = C - 2D$

$T''(0) = C > 0 \Rightarrow$ MIN AT $D = 0$

$T''(C) = C - 2C = -C < 0 \Rightarrow \boxed{\text{MAX AT } D = C}$

b) SENSITIVITY $= S = \frac{dT}{dD} = CD - D^2$

MAXIMIZE $S \Rightarrow$

$S' = \frac{d^2T}{dD^2} = C - 2D$

$0 = C - 2D$

$C = 2D$

$D = \frac{1}{2}C$

$S'' = -2 < 0 \Rightarrow \boxed{D = \frac{1}{2}C \text{ MAXIMIZES SENSITIVITY}}$

5. $x^2 + 9y^2 = 9 \quad \Rightarrow \quad 9y^2 = 9 - x^2 \quad \Rightarrow \quad y^2 = 1 - \frac{1}{9}x^2$

a) CLOSEST TO $(2, 0)$

$$S^2 = (y_2 - y_1)^2 + (x_2 - x_1)^2 \qquad\qquad -3 \leq x \leq 3$$

$$S = \left(0 - \sqrt{1 - \tfrac{1}{9}x^2}\right)^2 + (2 - x)^2$$

$$S = \left|1 - \tfrac{1}{9}x^2\right| + 4 - 4x + x^2$$

$$S' = \frac{-2}{9}x - 4 + 2x = \frac{16}{9}x - 4$$

$$0 = \frac{16}{9}x - 4$$

$$4 = \frac{16}{9}x$$

$$x = \frac{9 \cdot 4}{16} = \frac{9}{4}$$

$S'' = \frac{16}{9} > 0 \Rightarrow x = \frac{9}{4}$ GIVES A MINIMUM DISTANCE

$$x = \frac{9}{4}, \quad y = \pm\sqrt{1 - \tfrac{1}{9}\left(\tfrac{9}{4}\right)^2} = \pm\sqrt{1 - \tfrac{9}{16}} = \pm\sqrt{\tfrac{7}{9}} = \frac{\pm\sqrt{7}}{3}$$

$$\boxed{\left(\frac{9}{4}, \frac{\sqrt{7}}{3}\right) + \left(\frac{9}{4}, \frac{-\sqrt{7}}{3}\right)}$$

b) CLOSEST TO $(\sqrt{80}, 0)$

$$-3 \leq x \leq 3$$

$$S = 1 - \tfrac{1}{9}x^2 + (\sqrt{80} - x)^2 = 1 - \tfrac{1}{9}x^2 + 80 - 2\sqrt{80}x + x^2$$

$$S' = \frac{-2}{9}x - 2\sqrt{80} + 2x = \frac{16}{9}x - 8\sqrt{5}$$

$$0 = \frac{16}{9}x - 8\sqrt{5}$$

$$8\sqrt{5} = \frac{16}{9}x$$

$$x = \frac{9}{16}(8\sqrt{5}) = \frac{9\sqrt{5}}{2}$$

$S'' = \frac{16}{9} > 0 \Rightarrow$ MIN AX $x = \frac{9\sqrt{5}}{2}$ BUT $\frac{9\sqrt{5}}{2} > 3 \Rightarrow x = 3$ GIVES MI

$$x = 3 \qquad y = \pm\sqrt{1 - \tfrac{1}{9}(3)^2} = 0$$

$$\boxed{(3, 0)}$$

6. a) $f(x) = x^3 - 2x^2 - 3x + 1$

$f'(x) = 3x^2 - 4x - 3$

$0 = 3x^2 - 4x - 3$

$$x = \frac{4 \pm \sqrt{16 + 36}}{6} = \frac{4 \pm \sqrt{52}}{6} = \frac{4 \pm \sqrt{2 \cdot 21}}{6}$$

$f''(x) = 6x - 4$

$0 = 6x - 4$

$6x = 4$

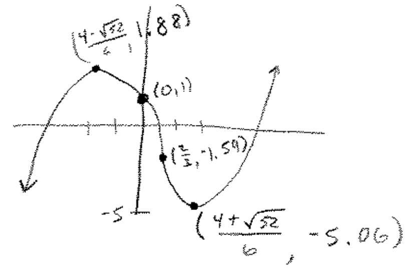
$x = \frac{2}{3}$ Pt Of Inflection

$f''\left(\frac{4+\sqrt{52}}{6}\right) = 6\left(\frac{4+\sqrt{52}}{6}\right) - 4 > 0 \Rightarrow$ Min

$f''\left(\frac{4-\sqrt{52}}{6}\right) = 6\left(\frac{4-\sqrt{52}}{6}\right) - 4 < 0 \Rightarrow$ Max

$-$ $+$

0 $\frac{2}{3}$ 10




b) $g(x) = (x+1)^4$

$g'(x) = 4(x+1)^3$

$0 = 4(x+1)^3$

$x = -1$

$-$ $+$

$-10$ $-1$ $10$

Min At $x = -1$

$g''(x) = 12(x+1)^2$

$0 = 12(x+1)^2$

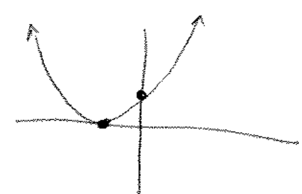
$x = -1$

$+$ $+$

$-10$ $-1$ $10$

$g(0) = 1$

$g(-1) = 0$



c) $h(x) = \frac{x^2 - x + 3}{x - 1}$ $\quad x \neq 1$

$$h'(x) = \frac{(2x-1)(x-1) - (x^2 - x + 3)}{(x-1)^2}$$

$$h'(x) = \frac{2x^2 - 3x + 1 - x^2 + x - 3}{(x-1)^2}$$

$$h'(x) = \frac{x^2 - 2x - 2}{(x-1)^2}$$

$0 = x^2 - 2x - 2$

$$x = \frac{2 \pm \sqrt{4 - 4(-2)}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

$+$ $-$ $-$ $+$

0 1

$$h''(x) = \frac{(2x-2)(x-1)^2 - (x^2 - 2x - 2)(2)(x-1)}{(x-1)^4}$$

$$h''(x) = \frac{(x-1)\left((2x-2)^{(x-1)} - 2x^2 + 4x + 4\right)}{(x-1)^4}$$

$$h''(x) = \frac{2x^2 - 3x + 2 - 2x^2 + 4x + 4}{(x-1)^3}$$

$$h''(x) = \frac{x+6}{(x-1)^3}$$

$0 = x + 6$

$x = -6$

$+$ $-$ $+$

$-6$ 1

$h(1+\sqrt{3}) = 4.464$

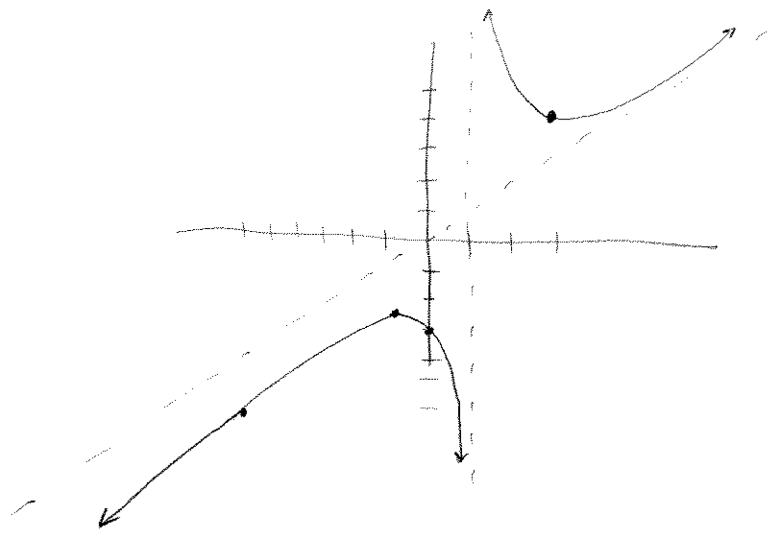
$h(1-\sqrt{3}) = -2.464$

$h(-6) = -6.428$

$$\lim_{x\to 1} \frac{x^2-x+3}{x-1} = \pm\infty$$

$$\lim_{x\to 1^+} (\quad) = +\infty$$

$$\lim_{x\to 1^-} (\quad) = -\infty$$



d) $s(t) = \frac{t^2}{t^2+4}$

$$s'(t) = \frac{2t(t^2+4) - 2t(t^2)}{(t^2+4)^2}$$

$$s'(t) = \frac{2t^3+8t-2t^3}{(t^2+4)^2}$$

$$s'(t) = \frac{8t}{(t^2+4)^2}$$

$0 = 8t$

$t=0$

$-$ | $+$

$0$

$$\lim_{t\to\infty} \frac{t^2}{t^2+4} = 1$$

$$s''(t) = \frac{8(t^2+4)^2 - 8t(2)(t^2+4)(2t)}{(t^2+4)^4}$$

$$= \frac{(t^2+4)(8t^2+32-32t^2)}{(t^2+4)^4}$$

$$= \frac{(t^2+4)(32-24t^2)}{(t^2+4)^4}$$

$0 = (t^2+4)(32-24t^2)$

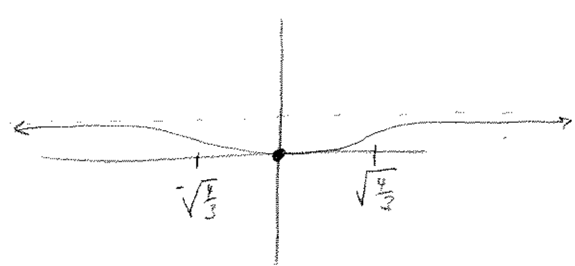
$32 = 24t^2$

$t^2 = \frac{32}{24} = \frac{8}{6} = \frac{4}{3}$

$t = \pm\sqrt{\frac{4}{3}}$

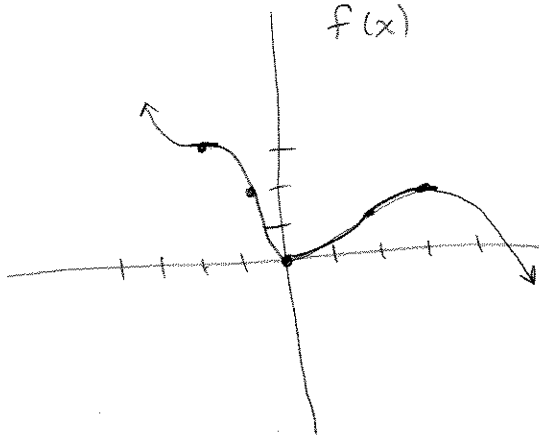
$-$ | $+$ | $-$

$-\sqrt{\frac{4}{3}}$ $\quad$ $\sqrt{\frac{4}{3}}$

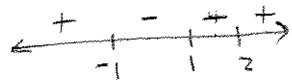


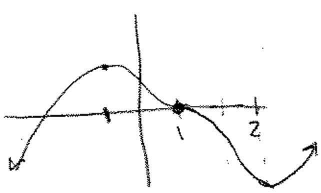
8. f IS EVERYWHERE CONTINUOUS

$f(-2)=3, \quad f(0)=0, \quad f(3)=2$

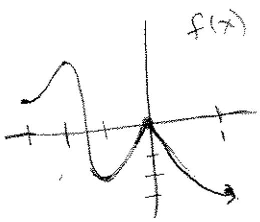



9. $f'(x)=(x-2)(x-1)^2(x+1) \qquad f(1)=0$




10.




11. ROLLE'S THRM: IF f IS CONTINUOUS ON $[a,b]$ AND DIFFERENTIABLE ON $(a,b)$, AND IF $f(a)=f(b)$ $\exists c \in (a,b)$ s.t. $f'(c)=0$.

Pf: BY THE MEAN VALUE THEOREM, WE KNOW $\exists c \in (a,b)$ s.t.

$$f'(c)=\frac{f(b)-f(a)}{b-a}=\frac{0}{b-a}=0. \quad \blacksquare$$

12. AT APPROX $\frac{3}{4}$ + $\frac{7}{3}$

13. $S(t)$ = DISTANCE BETWEEN HORSES AT TIME $t$.

$S(0) = S(\text{END})$

SO BY ROLLE'S THRM $\exists\, c$ ~~BETWEEN~~ A TIME IN THE RACE WHEN

$\frac{ds}{dt} = 0 \Rightarrow$ DISTANCE BETWEEN HORSES WAS NOT CHANGING $\Rightarrow$ VELOCITY OF HORSES

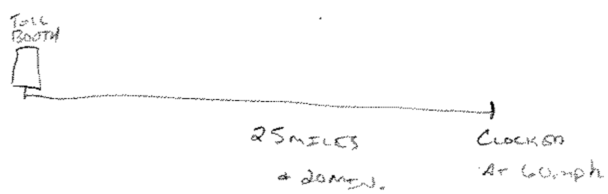
WAS THE SAME AT $c$.

OR $S(t) = d_1 - d_2$

$S'(t) = v_1 - v_2 = 0$ AT $c$

$\Rightarrow v_1 = v_2$ AT $c$.

14.




$\frac{25}{20} = \frac{5}{4}$ mi/min $\cdot \frac{60}{1\,hr} = 5(15) = 75$ = AVG. VELOCITY.

SO $\exists$ A POINT $c$ BETWEEN TOLL BOOTH + 25 MILES LATER

WHERE $v = 75$mph $\Rightarrow$ RENZO WAS SPEEDING.