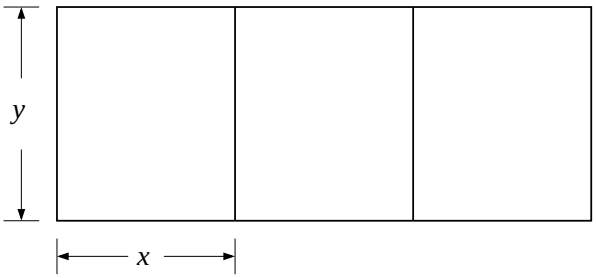
Math 1210-1 HW 9

Due Tuesday March 9, 2004

Please show all of your work and box your answer. Be sure to write in complete sentences when appropriate.

More Min-Max Problems

1. A farmer wishes to fence off three identical adjoining rectangular pens each with 16 square meters of area. What should the width, x, and length, y, of each pen be in order to minimize the amount of fence required?



2. Suppose that the outer boundary of the pens in Problem 1 requires heavy fence that costs \$10 per meter, but that the two internal partitions requre fence which costs only \$5 per meter. What dimensions x and y yield the least expensive pens?

3. When you cough, your windpipe contracts. The velocity, v, with which air comes out depends on the radius, r, of your windpipe. If R is the normal (rest) radius of your windpipe, then for $r \leq R$, the velocity is given by

$$v = a(R - r)r^2,$$

where a is a positive constant. What value of r maximizes the velocity?

4. The temperature change, T, in a patient generated by a dose, D of a particular drug is given by

$$T = \left(\frac{C}{2} - \frac{D}{3}\right) D^2,$$

where C is a positive constant.

   (a) What dose maximizes the temperature change?

   (b) The sensitivity of the body, at dosage D, to the drug is defined to be $\dfrac{dT}{dD}$. What dosage maximizes sensitivity?

5. Find the points on the ellipse

$$x^2 + 9y^2 = 9$$

   (a) Closest to the point $(2, 0)$.

   (b) Closest to the focus $(\sqrt{80}, 0)$.

   (Hint: Minimize the *square* of the distance. This avoids square roots.)

Sophisticated Graphing

6. Analyze the following functions and sketch their graphs:

   (a) $f(x) = x^3 - 2x^2 - 3x + 1$

   (b) $g(x) = (x+1)^4$

   (c) $h(x) = \dfrac{x^2 - x + 3}{x - 1}$

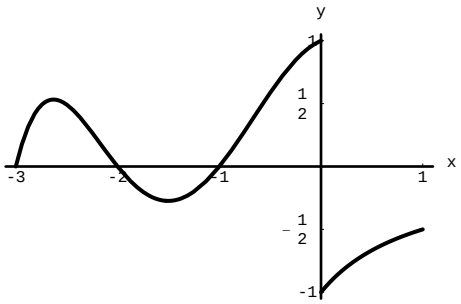
   (d) $s(t) = \dfrac{t^2}{t^2 + 4}$

7. Sketch the graph of the function f that has the following properties:

   - f is everywhere continuous,
   - $f(-2) = 3, f(0) = 0, f(3) = 2$,
   - $f'(-2) = 0, f'(3) = 0, f'(x) < 0$ for $x < -2$, $f'(x) < 0$ for $-2 < x < 0$, $f'(x) > 0$ for $0 < x < 3$, $f'(x) < 0$ for $x > 3$,
   - $f''(-2) = f''(-1) = f''(2) = 0$,
   - $f''(x) > 0$ for $x < -2$ and for $-1 < x < 2$,
   - $f''(x) < 0$ for $-2 < x < -1$ and for $2 < x$.

8. Suppose $f'(x) = (x-2)(x-1)^2(x+1)$ and $f(1) = 0$. Sketch a possible graph for f.

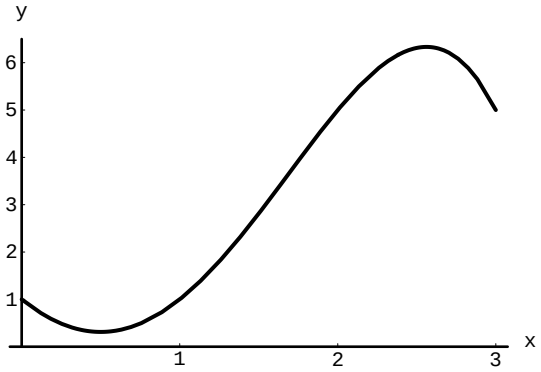
9. Let f be a continuous function with $f(0) = 0$ and $f(1) = -3$. If the graph of $y = f'(x)$ is as shown below, sketch a possible graph for $y = f(x)$



The Mean Value Theorem

10. Prove **Rolle's Theorem**: If f is continuous on $[a, b]$ and differentiable on (a, b), and if $f(a) = f(b)$, then there is at least one number c in (a, b) so that $f'(c) = 0$.

11. For the function graphed below, find (approximately) all points c that satisfy the conclusion to the Mean Value Theorem for the interval $[0, 3]$.



12. Suppose we are watching a horse race between Abel's Fortune, and Buttercup. If the two horses start at the same point and finish in a dead heat, prove that their speeds where identical at some instant of the race. (Hint: Let $s(t)$ be the distance between the two horses at time t and use Rolle's Theorem).

13. Bobby is driving his car. He stops at a toll booth. Twenty minutes later at a point 25 miles down the road, Bobby is clocked by a police officer at 60 miles per hour. If the speed limit is 60 miles per hour, prove to the officer that Bobby was speeding somewhere between the toll booth and the officer.