HW 8
SOLUTIONS

1. a) $f(x) = x^2 + 2x + 2$, $-2 \le x \le 0$

$f'(x) = 2x + 2$

$2x + 2 = 0$

$2x = -2$

$x = -1$

$\boxed{C.P. = \{-2, -1, 0\}}$

$f(-2) = 2$

$f(-1) = 1$

$f(0) = 2$

$\boxed{\text{MIN VALUE} = 1 \quad \text{MAX VALUE} = 2}$

b) $g(x) = x^2 - 3x + 2$, $-3 \le x \le 3$

$g'(x) = 2x - 3$

$2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

$g(-3) = 9 + 9 + 2 = 20$

$g(\frac{3}{2}) = \frac{9}{4} - \frac{9}{2} + 2 = \frac{9 - 18 + 8}{4} = \frac{-1}{4}$

$g(3) = 9 - 9 + 2 = 2$

$\boxed{C.P. = \{-3, \tfrac{3}{2}, 3\} \qquad \text{MIN VALUE} = \tfrac{-1}{4}, \ \text{MAX VALUE} = 20}$

c) $h(x) = \frac{1}{1+x}$, $0 \le x \le 4$

$h(x) = (1+x)^{-1}$

$h'(x) = -(1+x)^{-2} = \frac{-1}{(1+x)^2}$

$h'(x)$ D.N.E. AT $x = -1$

BUT $-1 < 0$

$\Rightarrow \boxed{C.P. = \{0, 4\}}$

$h(0) = \boxed{1 = \text{MAX VALUE}}$

$h(4) = \boxed{\frac{1}{5} = \text{MIN VALUE}}$

d) $s(t) = t^{2/3}$, $-4 \le t \le 4$

$s'(t) = \frac{2}{3} t^{-1/3}$

$s'(t) \ne 0$ BUT NOT DEFINED

AT $t = 0$

$s(-4) = (-4)^{2/3} = 2^{4/3}$

$s(0) = 0$

$s(4) = 2^{4/3}$

$\boxed{\text{SO } C.P. = \{-4, 0, 4\} \qquad \text{MIN VALUE} = 0 \quad \text{MAX VALUE} = 2^{4/3}}$

2. TWO POSITIVE #'s x, y s.t. $x + y = 23$ & $xy = \text{MAX}$.

IF $x + y = 23 \Rightarrow y = 23 - x$

SO $f(x) = x(23 - x) = 23x - x^2$

$f'(x) = 23 - 2x$

$23 - 2x = 0$

$C.P. = \{0, \frac{23}{2}, 23\}$

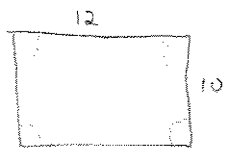
$f(0) = 0$

$f(\frac{23}{2}) = \frac{23}{2}(23 - \frac{23}{2}) = \frac{23}{2}$

$f(23) = 0$

$\Rightarrow \boxed{x = \frac{23}{2}, \ y = \frac{23}{2}}$

3. $V = ?$ $\quad 10 \times 12$

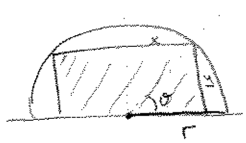


$$V(x) = (12-2x)(10-2x)x$$
$$= 2(6-x)2(5-x)x$$
$$= 4(6-x)(5-x)x$$
$$= 4(30-11x+x^2)x$$
$$= 120x - 44x^2 + 4x^3$$
$$V'(x) = 120 - 88x + 12x^2$$
$$120 - 88x + 12x^2 = 0$$
$$x = \frac{88 \pm \sqrt{88^2 - 4(12)(120)}}{2(12)} = \frac{88 \pm \sqrt{1984}}{24} = \frac{88 \pm 8\sqrt{31}}{24}$$
$$= \frac{8(11 \pm \sqrt{31})}{24} = \frac{11 \pm \sqrt{31}}{3}$$

$$C.P. = \left\{0, \cancel{\frac{11+\sqrt{31}}{3}}, \frac{11-\sqrt{31}}{3}, 5\right\}$$
≈ 5.5

$$V\left(\frac{11-\sqrt{31}}{3}\right) = \boxed{96.77 = \text{MAX VOLUME}}$$

4.



$$A = 2xy$$
$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$A = 2r^2\cos\theta\sin\theta$$

$$\frac{dA}{d\theta} = 2r^2(-\sin^2\theta + \cos^2\theta)$$

$$2r^2(-\sin^2\theta + \cos^2\theta) = 0$$
$$\cos^2\theta = \sin^2\theta$$
$$1 - \sin^2\theta = \sin^2\theta$$
$$1 = 2\sin^2\theta$$
$$\frac{1}{2} = \sin^2\theta$$
$$\cancel{\pm}\frac{1}{\sqrt{2}} = \sin\theta$$
$$\Rightarrow \theta = \frac{\pi}{4} \Rightarrow \boxed{x = \frac{r\sqrt{2}}{2} \quad y = \frac{r\sqrt{2}}{2}}$$

5. $f(x) = x^3 - x^2 - x + 1$

$f'(x) = 3x^2 - 2x - 1$

$0 = 3x^2 - 2x - 1$

$0 = (3x+1)(x-1)$

$x = 1, \frac{-1}{3}$

$+ \quad - \quad +$; $\frac{-1}{3} \quad 1$

$f''(x) = 6x - 2$

$6x - 2 = 0$

$6x = 2$

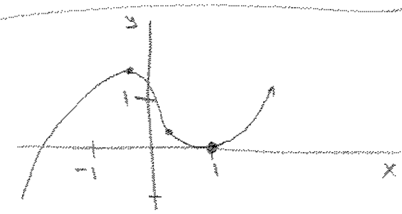
$x = \frac{1}{3}$

$- \quad +$; $\frac{1}{3}$

$f(-\frac{1}{3}) = \frac{-1}{27} - \frac{1}{9} + \frac{1}{3} + 1 = \frac{-1-3+9+27}{27} = \frac{32}{27}$

$f(1) = 0$

$f(\frac{1}{3}) = \frac{1}{27} - \frac{1}{9} - \frac{1}{3} + 1 = \frac{1-3-9+27}{27} = \frac{16}{27}$



C.P. $= \{1, -\frac{1}{3}, \frac{1}{3}\}$

GRAPH INCREASING ON $(-\infty, \frac{-1}{3}]$ + $(1, \infty)$. DECREASING ON $(\frac{-1}{3}, 1)$

CONCAVE UP ON $(\frac{1}{3}, \infty)$ CONCAVE DOWN ON $(-\infty, \frac{1}{3})$

b) $g(x) = x^4 + x^2 + 1$

$g'(x) = 4x^3 + 2x$

$4x^3 + 2x = 0$

$2x(2x^2+1) = 0$

$x = 0 \quad \cancel{2x^2 = -1}$

$- \quad +$; 0

$g''(x) = 12x^2 + 2$

$12x^2 + 2 = 0$

$12x^2 = -2$

$\cancel{x^2 =}$

$g(0) = 1$

ALWAYS CONCAVE UP

DECREASING ON $(-\infty, 0)$

INCREASING ON $(0, \infty)$

c) $h(x) = \frac{1}{x^2+1} = (x^2+1)^{-1}$

$h'(x) = \frac{-2x}{(x^2+1)^2}$

C.P. $x = 0$,

$+ \quad -$; 0

$h''(x) = \frac{-2(x^2+1)^2 + 2x(2)(x^2+1)(2x)}{(x^2+1)^4}$

$= \frac{-2(x^2+1)^2 + 8x^2(x^2+1)}{(x^2+1)^4}$

$0 = -2(x^2+1)^2 + 8x^2(x^2+1)$

$0 = (x^2+1)(-2(x^2+1) + 8x^2)$

$(x^2+1)(-2x^2 - 2 + 8x^2)$

$0 = (x^2+1)(6x^2 - 2)$

$6x^2 = 2$

$x^2 = \frac{1}{3}$

$x = \pm\frac{1}{\sqrt{3}}$

$+ \quad - \quad +$; $-\frac{1}{3} \quad \frac{1}{3}$

$h(0) = 1$

$h(-\frac{1}{3}) = \frac{1}{\frac{1}{9}+1} = \frac{1}{\frac{10}{9}} = \frac{9}{10}$

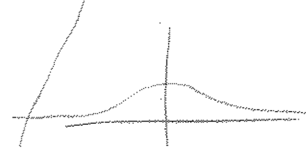
$h(\frac{1}{3}) = \frac{9}{10}$

INCREASING ON $(-\infty, 0)$

DECREASING ON $(0, \infty)$

CONCAVE UP ON $(-\infty, \frac{-1}{3}) + (\frac{1}{3}, \infty)$

CONCAVE DOWN ON $(\frac{-1}{3}, \frac{1}{3})$



d) $K(x) = \sin^2 x$ On $[0, 2\pi]$

$K'(x) = 2\sin x \cos x$

$0 = 2\sin x \cos x$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$+ \quad - \quad + \quad -$

$0 \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2}$

$K(0) = 0$

$K(\frac{\pi}{2}) = 1$

$K(\pi) = 0$

$K(\frac{3\pi}{2}) = 1$

$K(\frac{5\pi}{4}) = \frac{1}{2}$

$K(\frac{7\pi}{4}) = \frac{1}{2}$

$K''(x) = 2(\cos^2 x - \sin^2 x)$

$0 = 2(\cos^2 x - \sin^2 x)$

$1 - \sin^2 x = \sin^2 x$

$1 = 2\sin^2 x$

$-1 + 2\cos^2 x = 0$

$\cos x = \pm\frac{1}{\sqrt{2}}$

$\pm\frac{1}{\sqrt{2}} = \sin x$

$x = \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

$+ \quad - \quad + \quad - \quad +$

$\frac{\pi}{4} \quad \frac{3\pi}{4} \quad \frac{5\pi}{4} \quad \frac{7\pi}{4}$

INCREASING ON

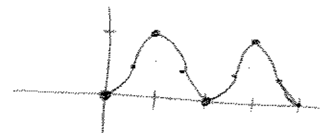
$(0, \frac{\pi}{2}), (\pi, \frac{3\pi}{2})$

DECREASING ON

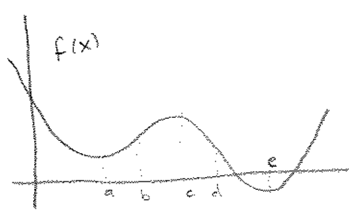
$(\frac{\pi}{2}, \pi), (\frac{3\pi}{2}, 2\pi)$

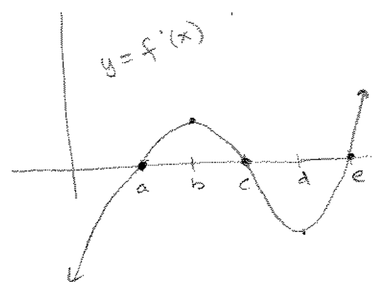
CONCAVE UP ON $(0, \frac{\pi}{4})$ + $(\frac{7\pi}{4}, 2\pi)$ + $(\frac{3\pi}{4}, \frac{5\pi}{4})$

CONCAVE DOWN ON $(\frac{5\pi}{4}, \frac{7\pi}{4})$ + $(\frac{\pi}{4}, \frac{3\pi}{4})$



6.



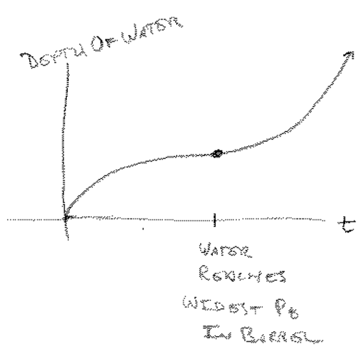


b) $f'(x)$ CHANGES SIGN AT a, c, + e

c) $f'(x)$ HAS LOCAL EXTREMA AT b + d.

7. THE LOCAL EXTREMA OF f POSSIBLY OCCUR WHEN $f'(x) = 0$. f CHANGES CONCAVITY WHEN $f''(x) = 0$. $f'(x)$ CHANGES SIGN AT LOCAL EXTREMA OF f. AND LOCAL EXTREMA OF $f'(x)$ OCCUR WHEN $f''(x) = 0$.

8.



9.

a) $f(x) = x^3 - 4x$

$f'(x) = 3x^2 - 4$

$0 = 3x^2 - 4$

$3x^2 = 4$

$x^2 = \frac{4}{3}$

$x = \pm \frac{2\sqrt{3}}{3}$

$f''(x) = 6x$

$6x = 0$

$x = 0$

$\boxed{C.P. = \left\{ \frac{-2\sqrt{3}}{3}, 0, \frac{2\sqrt{3}}{3} \right\}}$

$f''\left(\frac{2\sqrt{3}}{3}\right) > 0 \Rightarrow$ LOCAL MIN

$f''\left(\frac{-2\sqrt{3}}{3}\right) < 0 \Rightarrow$ LOCAL MAX

LOCAL MAX $= f\left(\frac{-2\sqrt{3}}{3}\right) = 3.0792$

LOCAL MIN $= f\left(\frac{2\sqrt{3}}{3}\right) = -3.0792$

b) $g(x) = x^4 + x^2 + 1$

$g'(x) = 4x^3 + 2x$

$0 = 4x^3 + 2x$

$0 = 2x(2x^2 + 1)$

$x = 0$

$g''(x) = 12x^2 + 2$

$0 = 12x^2 + 2$

$\frac{-1}{6} \cancel{=} x^2$

$g''(0) > 0 \Rightarrow$ LOCAL MIN

C.P. $= \{0\}$ LOCAL MIN IS $g(0) = 1$

c) $s(t) = t + \frac{1}{t}, \quad t \neq 0$

$s'(t) = 1 - \frac{1}{t^2}$

$0 = 1 - \frac{1}{t^2}$

$1 = \frac{1}{t^2}$

$t^2 = 1$

$t = \pm 1$

$s''(t) = \frac{2}{t^3}$

$t = 0$

$\Rightarrow s''(0)$ D.N.E.

BUT $0 \notin$ DOMAIN

$\boxed{C.P. = \{-1, \cancel{0}, 1\}}$

$s(-1) = -2$ LOCAL MIN

$s(1) = 2$ LOCAL MAX

~~$S(0)$~~

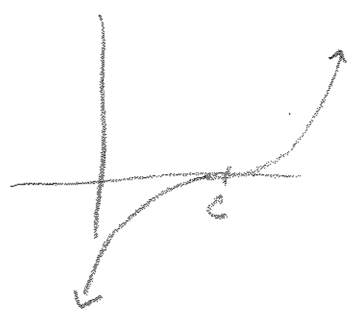
10. $f(c) = f'(c) = f''(c) = 0$ + $f'''(c) > 0$

So $f(c) = 0$, TANGENT LINE AT c TO f IS HORIZONTAL

AND c IS AN INFLECTION POINT, SINCE $f'''(c) > 0$

$\Rightarrow$ f CHANGES FROM CONCAVE DOWN TO CONCAVE UP

SO f COULD LOOK LIKE



Ex: $y = (x-3)^3$

$y' = 3(x-3)^2$ $\quad$ $y'' = 6(x-3)$ $\quad$ $y''' = 6$

$y(3) = 0$

$y'(3) = 0$

$y''(3) = 0$

$y'''(3) > 0$