

Math 1210-1 HW 8
Due Wednesday March 3, 2004

Please show all of your work and box your answer. Be sure to write in complete sentences when appropriate.

Maxima and Minima

1. Identify the critical points and find the maximum value and the minimum value:

(a) $f(x) = x^2 + 2x + 2, -2 \leq x \leq 0.$

(b) $g(x) = x^2 - 3x + 2, -3 \leq x \leq 3.$

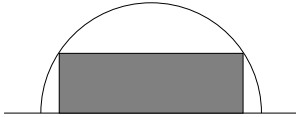
(c) $h(x) = \frac{1}{1+x}, 0 \leq x \leq 4.$

(d) $s(t) = t^{2/3}, -4 \leq t \leq 4.$

2. Find two positive numbers whose sum is 23 and whose product is a maximum.

3. Find the volume of the largest open box that can be made from a piece of cardboard with width 10 and length 12 by cutting squares from the corners and turning up the sides.

4. A rectangle is to be inscribed in a semicircle of radius r , as shown below. What are the dimensions of the rectangle if its area is to be maximized?



Monotonicity and Concavity

5. Determine where the graph of the given function is increasing, decreasing, concave up, and concave down. Then sketch the graph of the function.

(a) $f(x) = x^3 - x^2 - x + 1.$

(b) $g(x) = x^4 + x^2 + 1.$

(c) $h(x) = \frac{1}{x^2 + 1}.$

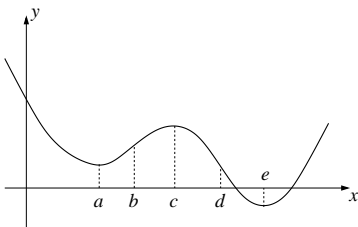
(d) $k(x) = \sin^2 x$ on $[0, 2\pi].$

6. For the function f whose graph $y = f(x)$ is given below,

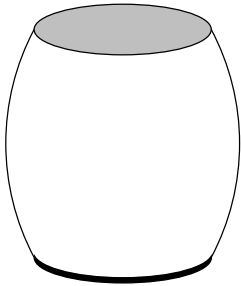
(a) Sketch $f'(x)$

(b) Where does $f'(x)$ change its sign?

(c) Where does $f'(x)$ have a local extremum (i.e., maximum or minimum)?



7. Using Problem 6 as a guide, write a short paragraph (using complete sentences) which describes the relationship between the following features of a function f (an arbitrary function):
- The local extrema of f ,
 - The points at which f changes concavity,
 - The sign changes of f' ,
 - The local extrema of f' ,
8. Water is flowing at a constant rate (i.e., constant volume flow per unit time) into the barrel shown below. Sketch a graph of the depth of the water against time. Mark on the graph the time at which the water reaches the widest point of the barrel.



Local Maxima and Minima

9. Find all critical points for the following functions, and determine the local extremal values (local maxima and local minima).
- (a) $f(x) = x^3 - 4x$
- (b) $g(x) = x^4 + x^2 + 1$
- (c) $s(t) = t + \frac{1}{t}$, $t \neq 0$
10. Suppose f is a function with $f(c) = f'(c) = f''(c) = 0$ and $f'''(c) > 0$. What can you conclude about f ? Sketch a possible graph for f near c .