Math 1210-1 HW 6

Due Wednesday February 18, 2004

Please show all of your work and box your answer. Be sure to write in complete sentences when appropriate. Also, I prefer you to leave answers like $\sqrt{2}$ in that form, rather than in decimal form like 1.414. Note that the symbol ⊞ indicates that graph paper might be useful for that problem.

Rules for Finding Derivatives

1. We learned that the derivative is a *linear operator*. Find another simple linear operator L and show that L is a linear operator. Hint: Remember that operators take functions as their inputs and output functions as well. Keep it simple.

2. Find $D_x y$ using the rules we have learned.

   (a) $y = 3x^2$

   (b) $y = kx$

   (c) $y = x^9 + x^7 + x^5 + x^3 + x + 1$

   (d) $y = \dfrac{1}{x} + x^2$

   (e) $y = \dfrac{\pi}{x^3}$

   (f) $y = (x^2 + x + 1)(x - 1)$

   (g) $y = \dfrac{x^2 + x + 1}{x - 1}$

   (h) $y = 17x^3 \left(\dfrac{x + 1}{x^2 - x + 1} \right)$

   (i) $y = \dfrac{3}{23x^2} - \dfrac{13}{2}$

   (j) $y = 2x^{-5} + 3x^{-3} + x^{-1}$

   (k) $y = x^{987654321}$

   (l) $y = (x - 4)^2$

   (m) $y = (x^2 + 1)^3$

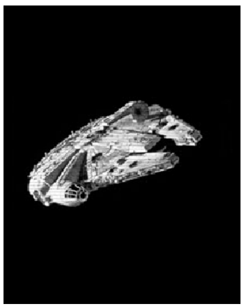
3. Use the Product Rule to show each of the following:

   (a) $D_x [f(x)]^2 = 2 \cdot f(x) \cdot D_x f(x)$.

   (b) $D_x [f(x)]^3 = 3 \cdot [f(x)]^2 \cdot D_x f(x)$.

   (c) What do you think $D_x [f(x)]^n$ (where n is a positive integer) will be?

4. Suppose $f(1) = 1, f'(1) = 2, g(1) = 3, g'(1) = 4$. Find
   (a) $(f + g)'(1)$
   (b) $(f \cdot g)'(1)$
   (c) $(f/g)'(1)$
5. Prove the Difference Rule.
6. Find all of the points on the graph $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 + 1$ where the tangent line is horizontal.
7. Chewbacca is flying the Millenium Falcon along the curve $y = x^2$. If the wookie turns off his ion engines, the Falcon will fly off along a tangent line to this curve. He is almost out of power when he notices that a station on Yavin V is open at the point with coordinates $(5, 24)$. Making quick calculations, Chewie is able the determine the point (x, y) at which he should cut his engines (all wookies take calculus). What is the point he determined?



8. Find $x > 0$ so that the tangent lines to the graphs $y = x^2$ and $y = -x^2 + \dfrac{3}{2}x + 1$ at x are perpendicular.

Derivatives for Trigonometric Functions

9. Find $D_x y$.
   (a) $y = 3\sin x + 2\cos x$
   (b) $y = \sin^3 x$
   (c) $y = \tan x + \sin x$
   (d) $y = (\sin x)(\cos x)$
   (e) $y = \dfrac{\sin x + \cos x}{\sin x - \cos x}$
   (f) $y = x\sin x$
   (g) $y = x^3 \cos x$
   (h) $y = \dfrac{x\sin x + \cos x}{x^2 + 1}$
   (i) $y = \sin^2 x + \cos^2 x$
   (j) $y = \cos x - x$
10. Use the definition of derivative to find $D_x(\cos 3x)$.
11. Find the equation of the tangent line to $y = \cos x$ at $x = \dfrac{\pi}{2}$.
12. A Ferris wheel of radius 10 meters is rotating counterclockwise at an angular velocity of 1 radian per second. At time $t = 0$, one seat on the wheel has coordinates $(10, 0)$.
    (a) What are the coordinates of this seat at time t?
    (b) How fast is it rising (vertically) at time t?
    (c) When is it rising most quickly? What is its vertical speed at this time?

The Chain Rule

13. Find the derivative of each of the following functions:

    (a) $f(x) = (x+1)^{99}$
    (b) $f(x) = \sin(x^2)$
    (c) $w = (t^2+1)^{100}$
    (d) $w = (t^3+1)^{100}$
    (e) $f(x) = x \sin\left(\frac{1}{x}\right)$
    (f) $g(t) = \tan(\cos t)$
    (g) $f(x) = \sin^2 x$
    (h) $h(t) = \dfrac{1}{\sin^2(t)}$
    (i) $y = \sin^3(x^2+x+1)$
    (j) $z = \cos^n x$, where n is a positive integer.
    (k) $w = x^2 \sin(x^2)$

14. Given the following information:

$$\begin{aligned} f(2) &= 1 & g(4) &= 2 \\ f(4) &= 3 & g(3) &= 4 \\ f'(2) &= 5 & g'(4) &= 6 \\ f'(4) &= 7 & g'(3) &= 8. \end{aligned}$$

    Find:

    (a) $h(4)$ if $h(x) = f(g(x))$
    (b) $h'(4)$ if $h(x) = f(g(x))$
    (c) $h(4)$ if $h(x) = g(f(x))$
    (d) $h'(4)$ if $h(x) = g(f(x))$
    (e) $h'(4)$ if $h(x) = \dfrac{f(x)}{g(x)}$.

15. A pebble is dropped into still water, forming a circular ripple whose radius is expanding at a constant rate of 10 centimeters per second. Find a formula for the area enclosed by the ripple as a function of time. When the radius is 20 cm, how fast is the area enclosed by the ripple increasing?

16. Use the Chain Rule to show that the derivative of an odd function is even and that the derivative of an even function is odd.