HW 5 Solutions

1. $S(t) = 5t^2$

a) $S(1) - S(0) = 5 - 0 = \boxed{5\,m}$

b) $S(2) - S(1) = 5(2)^2 - 5(1)^2 = 20 - 5 = \boxed{15\,m}$

c) $$V_{AVG} = \frac{S(5) - S(4)}{5-4} = \frac{125 - 80}{1} = \boxed{45\,m/s}$$

d) $$V_{AVG} = \frac{S(4.01) - S(4)}{4.01 - 4} = \frac{0.4005}{0.01} = \boxed{40.05\,m/s}$$

e) $$V = \lim_{h\to 0}\left(\frac{S(4+h) - S(4)}{h}\right) = \lim_{h\to 0}\frac{5(4+h)^2 - 80}{h} = \lim_{h\to 0}\frac{5(16+8h+h^2) - 80}{h}$$

$$= \lim_{h\to 0}\frac{h(40+h^2)}{h} = \boxed{40\,m/s}$$

2. a) $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h\to 0}\frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h\to 0}\frac{\sqrt{x+h} - \sqrt{x}}{h}\cdot\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0}\frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0}\frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

b) $f(x) = \frac{3}{x}$

$$f'(x) = \lim_{h\to 0}\frac{\frac{3}{x+h} - \frac{3}{x}}{h} = \lim_{h\to 0}\frac{\frac{3x - 3(x+h)}{x(x+h)}}{h} = \lim_{h\to 0}\frac{3x - 3x - 3h}{h(x)(x+h)}$$

$$= \lim_{h\to 0}\frac{-3h}{h(x)(x+h)} = \lim_{h\to 0}\frac{-3}{x(x+h)} = \frac{-3}{x(x)} = \boxed{\frac{-3}{x^2}}.$$

c) $f(x) = \frac{x}{x+1}$

$$f'(x) = \lim_{h\to 0}\frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h} = \lim_{h\to 0}\frac{(x+h)(x+1) - x(x+h+1)}{h(x+1)(x+h+1)} = \lim_{h\to 0}\frac{x^2+x+hx+h-x^2-xh-x}{h(x+1)(x+h+1)}$$

$$= \lim_{h\to 0}\frac{h}{h(x+1)(x+h+1)} = \lim_{h\to 0}\frac{1}{(x+1)(x+h+1)} = \frac{1}{(x+1)(x+1)} = \boxed{\frac{1}{(x+1)^2}}$$

3. a) $f'(0) \approx \boxed{0}$

$f'(2) \approx \frac{6-1}{4-1} = \boxed{\frac{5}{3}}$

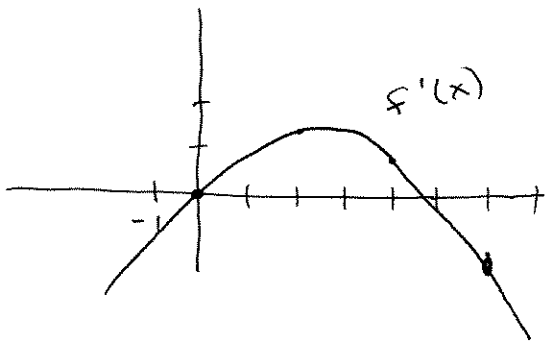
$f'(4) \approx \boxed{\frac{3}{4}}$

$f'(6) \approx \boxed{-\frac{3}{2}}$

b) x-COORDINATES OF VERTICES ARE $\boxed{x = 0}$ & $\boxed{x \approx 4.8}$

THE DERIVATIVE AT THESE TWO POINTS IS ZERO.

c)



4. f(x) IS THE SOLID LINE

f'(x) IS THE DOTTED LINE.