

Math 1210-1    Homework 2  
Due Wednesday January 21

Show all work. Please box your answers. Be sure to write in complete sentences when appropriate. Also, I prefer exact answers like  $\sqrt{2}$  instead of 1.414. Note that a symbol  $\boxplus$  indicates that graph paper might be useful for that problem.

## Functions and their Graphs

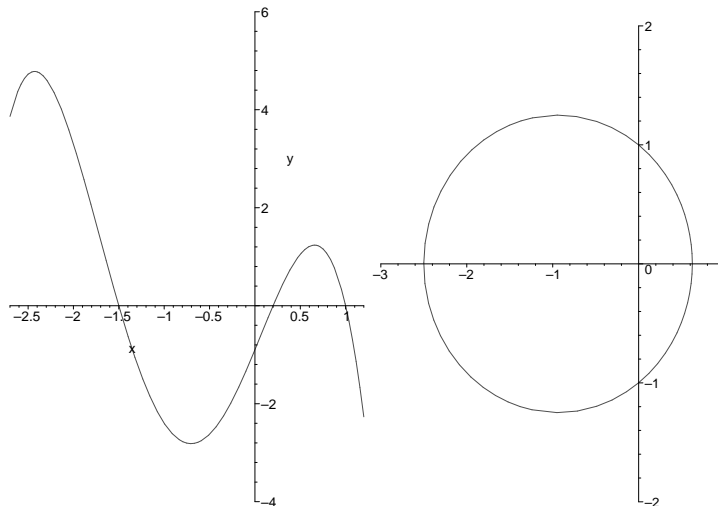
1. For  $f(x) = x^5 + 2x$  find each value. (Note that you should NOT use your calculator on this one; I don't want a decimal answer.)

- (a)  $f(1)$
- (b)  $f(\sqrt{2})$
- (c)  $f\left(\frac{1}{4}\right)$
- (d)  $f(\pi)$
- (e)  $f\left(\frac{1}{t}\right)$

2. Which of the following determine a function  $f$  with the formula  $y = f(x)$ ? For those that do, find  $f(x)$ . Note that the definition of a function requires a single  $y$  for each  $x$ .

- (a)  $9x^2 + 16y^2 = 25$
- (b)  $x = \sqrt{3y - 2}$
- (c)  $x = \frac{y}{y - 1}$
- (d)  $3y + x - xy = 1, x \neq 3$

3. Which of the following graphs are the graphs of functions  $y = f(x)$ ?



4. For  $f(t) = \frac{t}{t+1}$ , find and simplify  $\frac{f(t+h) - f(t)}{h}$ .

5.  $\boxplus$  State whether each function is even, odd, or neither, and then sketch its graph.

- (a)  $f(x) = 7$
- (b)  $g(x) = \pi x$

- (c)  $F(x) = 3x - 1$   
 (d)  $G(x) = |3x|$   
 (e)  $f(t) = \frac{1}{3}t^3$   
 (f)  $\phi(z) = \frac{z-1}{z+1}$
6. Let  $p$  denote the perimeter of an equilateral triangle. Find a formula for  $A(p)$ , the area of such a triangle.
7.  $\boxplus$  Let  $A(c)$  denote the area of the region bounded from above by the line  $y = 2x + 1$ , from the left by the  $y$ -axis, from below by the  $x$ -axis, and from the right by the line  $x = c$ . Such a function is called an accumulation function. We will see more accumulation functions later in the course. Find:
- (a)  $A(1)$   
 (b)  $A(0)$   
 (c)  $A(2)$   
 (d)  $A(c)$   
 (e) Sketch the graph of  $A(c)$ .  
 (f) What are the domain and range of  $A(c)$ ?
8. Which of the following functions satisfies  $f(x + y) = f(x) + f(y)$  for all  $x$  and  $y$  in  $\mathbb{R}$ ?
- (a)  $f(t) = t$   
 (b)  $f(t) = t^2$   
 (c)  $f(t) = t^3$   
 (d)  $f(t) = 2t$   
 (e)  $f(t) = t + 2$   
 (f)  $f(t) = -5t$
- Do you have any conjectures about what other functions have this property?
9. Suppose that  $f$  is a function with the property in Problem 8, namely  $f(x + y) = f(x) + f(y)$  for all  $x$  and  $y$  in  $\mathbb{R}$ . Prove that there is a number  $m$  such that  $f(t) = mt$  for all rational numbers  $t$ . Hint: first decide what  $m$  has to be. Then proceed in steps, starting with showing  $f(0) = 0$ ,  $f(p) = mp$  for  $p$  in  $\mathbb{N}$ ,  $f(\frac{1}{p}) = \frac{m}{p}$ , and so on.

## Operations on Functions

10. For  $f(x) = x^2 + x$  and  $g(x) = \frac{1}{x+1}$ , find each value:
- (a)  $(f - g)(2)$   
 (b)  $(f/g)(1)$   
 (c)  $(g^3)(3)$   
 (d)  $(f \circ g)(1)$   
 (e)  $(g \circ f)(1)$   
 (f)  $(g \circ g)(1)$
11. For  $f$  and  $g$  as in Problem 10, find a formula for each of the following:
- (a)  $(f \cdot g)(x)$   
 (b)  $f^2(x) + g^3(x)$   
 (c)  $(f \circ g)(x)$   
 (d)  $(g \circ g)(x)$

12. For each function  $F$  below, find functions  $f$  and  $g$  so that  $F = f \circ g$ .
- $F(x) = (x + 1)^3$
  - $F(x) = \log(3x + 2)$
  - $F(x) = \sqrt{x^2 - 1}$
13.  $\boxplus$  Sketch the graph of  $f(x) = \sqrt{x + 2} - 1$  by first sketching the graph of  $g(x) = \sqrt{x}$  and then translating.
14. State whether each of the following is an odd function, an even function, or neither. Prove your assertions.
- $f + g$  where both  $f$  and  $g$  are even.
  - $f + g$  where both  $f$  and  $g$  are odd.
  - $f \cdot g$  where both  $f$  and  $g$  are even.
  - $f \cdot g$  where both  $f$  and  $g$  are odd.
  - $f \cdot g$  where  $f$  is even and  $g$  is odd.
15. Let  $F$  be any function such that if  $x$  is in the domain of  $F$  then  $-x$  is in the domain of  $F$ . Prove each of the following:
- If  $g$  is the function given by  $g(x) = F(x) + F(-x)$ , then  $g$  is an even function.
  - If  $h$  is the function given by  $h(x) = F(x) - F(-x)$ , then  $h$  is an odd function.
  - $F$  can always be expressed as the sum of an even function and an odd function. (What are they?)

## The Trigonometric Functions

16. Convert each of the following degree measures to radians (leave  $\pi$  in your answer).
- $45^\circ$
  - $60^\circ$
  - $-30^\circ$
  - $18^\circ$
  - $270^\circ$
  - $-540^\circ$
  - $-10^\circ$
17. Evaluate without using a calculator:
- $\tan\left(\frac{\pi}{4}\right)$
  - $\tan(0)$
  - $\sec\left(\frac{\pi}{4}\right)$
  - $\csc\left(\frac{\pi}{6}\right)$
  - $\sin\left(-\frac{\pi}{3}\right)$
18.  $\boxplus$  Sketch the graphs of the following on  $[-\pi, 2\pi]$ .
- $y = 3 \cos x$
  - $y = \cos 3x$
  - $y = \sin\left(x - \frac{\pi}{4}\right)$

19.  $\boxplus$  For each of the following, determine the period, amplitude, and shifts (both horizontal and vertical) and draw a graph over the interval  $-5 \leq x \leq 5$ .

(a)  $y = 2 \cos \frac{x}{3}$

(b)  $y = \tan 2x$

(c)  $y = 4 \cos(x - \frac{\pi}{3}) + 1$

20. State whether each of the following functions is even, odd, or neither.

(a)  $\sin^2 x$

(b)  $\cos 2x$

(c)  $\sin x \cos x$

(d)  $\sin^3 x$

(e)  $\cos(\sin x)$

(f)  $x + \sin x$

21. Use the addition identities to find analogous identities for each of the following:

(a)  $\sin(x - y)$

(b)  $\cos(x - y)$

(c)  $\tan(x - y)$

22. Suppose the normal high temperature for Salt Lake City is  $5^\circ\text{F}$  for January 15 and  $105^\circ\text{F}$  for July 15. Assuming that these are the extreme high and low temperatures for the year, use this information to approximate the average high temperature for November 15.