Math 1210-1 Homework 1 Solutions

Straight Lines

1. $(0,1),(1,2)$ The slope of the line containing these two points is $m = \frac{\delta y}{\delta x} = \frac{2-1}{1-0} = 1$.
3. $(1,1),(3,2), m = \frac{2-1}{3-1} = \frac{1}{2}$
5. $(-2,3),(3,1), m = \frac{1-3}{3+2} = \frac{-2}{5}$
7. $m = 2, (0,0)$ corresponds to the line $y - 0 = 2(x - 0)$ or $y = 2x$.
9. $m = -3, (2,-1)$ corresponds to the line $y + 1 = -3(x - 2)$.
11. $m = \frac{-2}{3}, (0,5)$ corresponds to the line $y - 5 = \frac{-2}{3}x$.
13. $m = 3$, y-intercept $= 1$ corresponds to the line $y = 3x + 1$.
15. $6x - 2y = 4 \Longrightarrow 2y = 6x - 4 \Longrightarrow y = 3x - 2$, therefore the y-intercept is at -2, and the slope is 3.
17. $(1,1)$ parallel to $y = 3x + 2$. $m = 3$, so the parallel line would be $y - 1 = 3(x - 1)$.
19. $(1,0)$ perpendicular to $y = 3x + 2$. Perpendicular means the slope is the negative reciprocal, so $m = \frac{-1}{3}$ and the line is $y = \frac{-1}{3}(x - 1)$.
21. $(0,1)$ perpendicular to $x = 3$. This line is vertical, so the perpendicular line will be horizontal of the form $y = k$, and since the point $(0,1)$ is on the line, we know $y = 1$ is the perpendicular line to $x = 3$ at the point $(0,1)$.
23. The line perpendicular to the line $2y - x = 4$ at $(1,1)$ has slope $m = -2$, so the equation of the perpenducular line is $y - 1 = -2(x - 1)$ or $y = -2x + 3$. We want to find where the two lines intersect, so we solve these two equations and two unknowns by substitution or elimination or matricies. I prefer substitution for this problem. So $\frac{1}{2}x + 2 = -2x + 3$, and we solve for x and find $x = \frac{2}{5}$. Substituting that back into $y = -2(\frac{2}{5}) + 3$, we get $y = \frac{11}{5}$. Therefore the two lines intersect at the point $(\frac{2}{5}, \frac{11}{5})$. Using the distance formula with these two points, we get $d = \sqrt{(1 - \frac{2}{5})^2 + (1 - \frac{11}{5})^2} = \frac{3\sqrt{5}}{5}$.
25. We need a line perpendicular to $y = 2x$ and $y = 2x + 3$. We will choose the line perpendicular to $y = 2x$ at $(0,0)$, and it will also be perpendicular to the other line. This perpendicular line has the equation $y = \frac{-1}{2}x$. We need to find the point where the perpendicular line intersects $y = 2x + 3$. Solving these two equations as in the previous problem, we get $x = \frac{-6}{5}$ and $y = \frac{3}{5}$. Now we need the distance between $(0,0)$ and $(\frac{-6}{5}, \frac{3}{5})$. Using the distance formula we get $d = \sqrt{(\frac{-6}{5})^2 + (\frac{3}{5})^2} = \frac{3\sqrt{5}}{5}$.