

Math 1210-1 HW 11
Due Wednesday March 31, 2004

Please show all of your work and box your answer. Be sure to write in complete sentences when appropriate.

Differential Equations

1. For each of the following differential equations, find the general solution (the family of solutions parameterized by the constant C). Then find a particular solution which also satisfies the given condition.

(a) $\frac{dy}{dx} = x^2 + x$, $y = 1$ at $x = 1$

(b) $\frac{dy}{dx} = \frac{x}{y}$, $y = 2$ at $x = -2$

(c) $\frac{dy}{dx} = -\frac{x}{y}$, $y = 3$ at $x = 4$

(d) $\frac{dy}{dx} = \frac{\sin x}{\cos y}$, $y = \frac{\pi}{2}$ at $x = 0$

(e) $\frac{dy}{dt} = y^2(t^2 - 1)$, $y = 2$ at $t = 0$

(f) $\frac{d^2s}{dt^2} = s^3$, $s = 2$ at $t = 1$, $\frac{ds}{dt} = 1$ at $t = 1$

2. Find the xy -equation for the curve through $(0, 1)$ whose slope at any point is twice the square of its y -coordinate.

3. A projectile is thrown upward from a height s_0 above the surface of the Earth with a speed of v_0 . Let $s(t)$ be its height above the surface of the Earth at time t . Let g be the acceleration due to gravity. Write down a differential equation relating the acceleration of the projectile to acceleration due to gravity. Solve this differential equation with the initial conditions $s(0) = s_0$, and $\frac{ds}{dt}(0) = v_0$ to get the familiar formula from physics

$$s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$$

for the height s at time t .

4. Refer to Problem 3. A ball is thrown directly upwards from the ground with an initial speed of 20 meters per second. What is the maximum height the ball reaches? (Use the approximate value for the acceleration due to gravity $g = 9.81\text{m/sec}^2$).

Sums and Sigma Notation

5. Find the value of each of the following sums.

(a) $\sum_{n=1}^5 (n+1)$

(b) $\sum_{i=1}^7 i^2$

(c) $\sum_{k=0}^5 \frac{(-1)^k 2^k}{k+1}$

(d) $\sum_{n=0}^4 \frac{3^n}{n!}$, where $0! = 1$ and $n! = n[(n-1)!]$.

(e) $\sum_{n=0}^5 n \cos(n\pi)$

6. Write each of the following summations in sigma notation.

(a) $1 + 2 + 3 + \cdots + 27$

(b) $1 + 4 + 9 + 16 + \cdots + 625$

(c) $a_1 + \cdots + a_N$

(d) $a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$

(e) $b_1 + b_3 + b_5 + \cdots + b_{101}$

(f) $f(c_1) + f(c_2) + f(c_3) + \cdots + f(c_N)$

(g) $f(c_1)\Delta x + f(c_2)\Delta x + \cdots + f(c_N)\Delta x$

7. Suppose that $\sum_{i=1}^N a_i = A$ and $\sum_{i=1}^N b_i = B$. Find

(a) $\sum_{i=1}^N (a_i - b_i)$

(b) $\sum_{i=1}^N (2a_i + 3b_i)$

(c) $\sum_{i=1}^N (a_i + b_i + 1)$

8. Find the value of each of the following sums. (Hint: They collapse!)

(a) $\sum_{k=1}^{23} \left(\frac{1}{k} - \frac{1}{k+1} \right)$

(b) $\sum_{k=1}^{10} (A^k - A^{k-1})$, where A is some constant.

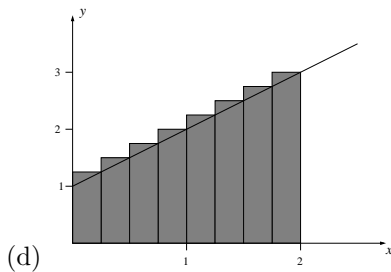
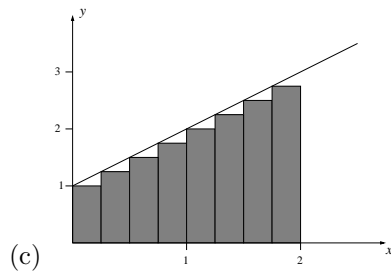
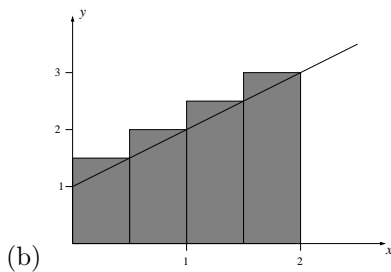
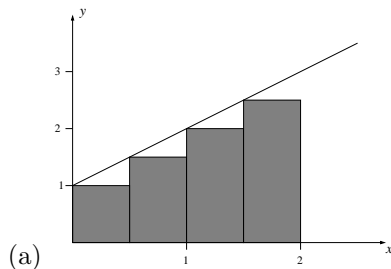
9. Prove the following formula for a **geometric sum**

$$\sum_{k=0}^N r^k = 1 + r + r^2 + \cdots + r^N = \frac{1 - r^{N+1}}{1 - r}$$

(Hint: Let $S = 1 + r + r^2 + \cdots + r^N$. Evaluate $S - rS$ and solve for S .)

Introduction to Area

10. Find the area of the indicated polygons on the graph $y = x + 1$ between $x = 0$ and $x = 2$.



11. In Problem 10, we had either four or eight rectangles either inscribed or circumscribed on the graph of $y = x + 1$ between $x = 0$ and $x = 2$. What would the area be if we had:

- 16 rectangles inscribed on the graph $y = x + 1$ between $x = 0$ and $x = 2$?
- 16 rectangles circumscribed on the graph $y = x + 1$ between $x = 0$ and $x = 2$?
- 2^n rectangles inscribed on the graph $y = x + 1$ between $x = 0$ and $x = 2$?
- 2^n rectangles circumscribed on the graph $y = x + 1$ between $x = 0$ and $x = 2$?
- What happens as $n \rightarrow \infty$?

12. Repeat Problem 11, but with the graph $y = x^2 + 1$ between $x = 0$ and $x = 2$.

13. Suppose that an object is traveling in a straight line, and its velocity at time t seconds is given by $v(t) = t + 1$. How far did the object travel between $t = 0$ and $t = 2$?

The Definite Integral

For this section you should use only the definition of Definite Integrals to calculate.

14. For each function f , interval I , and partition φ with sample points, calculate the Riemann sum

$$\sum_{i=1}^n f(\bar{x}_i) \Delta x_i.$$

- (a) $f(x) = x^2 + 1$, $I = [0, 2]$, φ : divide I into 10 subintervals, evaluate at the midpoint of each subinterval.
(b) $f(x) = x^3$, $I = [0, 1]$, φ : divide I into n subintervals, evaluate at the left-hand point of each subinterval. Is this a lower bound or an upper bound on the integral?
(c) $f(x) = x^3$, $I = [0, 1]$, φ : divide I into n subintervals, evaluate at the right-hand point of each subinterval. Is this a lower bound or an upper bound on the integral?

15. Use the formula

$$\sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2} \right]^2$$

and Problem 14 to calculate $\int_0^1 x^3 dx$.

16. Find $\int_0^4 x + 2 dx$ using the definition.

17. Recall that $[[x]]$ denotes the greatest integer less than or equal to x . Find $\int_0^4 [[x]] dx$.

18. Let f be an odd function and g be an even function. Furthermore, suppose

$$\int_0^2 |f(x)| dx = \int_0^2 g(x) dx = 5.$$

Use (and state) geometric reasoning to calculate each of the following:

(a) $\int_{-2}^2 f(x) dx$

(b) $\int_{-2}^2 g(x) dx$

(c) $\int_{-2}^2 |f(x)| dx$

(d) $\int_{-2}^2 f(x)g(x) dx$

19. State whether or not you think the following functions are integrable on every interval $[a, b]$. Justify your claim.

(a) $f(x) = x^{57} + x^{32} - 122372x + 1$

(b) $f(x) = \begin{cases} 0, & \text{if } x \text{ is an integer,} \\ x, & \text{otherwise.} \end{cases}$

(c) $f(x) = \begin{cases} 1, & \text{if } x \text{ is rational,} \\ 0, & \text{otherwise.} \end{cases}$