HW 10 SOLUTIONS

1. a) $\int 2\,dx = \boxed{2x + C}$

b) $\int x + \pi \, dx = \boxed{\frac{1}{2}x^2 + \pi x + C}$

c) $\int x^{1000} + x^2 \, dx = \boxed{\frac{1}{1001}x^{1001} + \frac{1}{3}x^3 + C}$

d) $\int \frac{x^5 - 2x}{x^3}\,dx = \int \frac{x^5}{x^3} - \frac{2x}{x^3}\,dx = \int x^2 - 2x^{-2}\,dx = \boxed{\frac{1}{3}x^3 + 2x^{-1} + C}$

e) $\int \theta + \cos\theta \, d\theta = \boxed{\frac{1}{2}\theta^2 + \sin\theta + C}$

f) $\int 4x^{5/2}\,dx = 4\left(\frac{2}{7}x^{7/2}\right) + C = \boxed{\frac{8}{7}x^{7/2} + C}$

g) $\int (\sqrt{3}\,x^2 + 1)^{11}\, 2\sqrt{3}\,x\,dx$

LET $u = \sqrt{3}\,x^2 + 1$

THEN $du = 2\sqrt{3}\,x\,dx \Rightarrow dx = \frac{du}{2\sqrt{3}\,x}$

$\int u^{11}\, 2\sqrt{3}\,x \frac{du}{2\sqrt{3}\,x} = \int u^{11}\,du = \frac{1}{12}u^{12} + C = \boxed{\frac{1}{12}(\sqrt{3}\,x^2 + 1)^{12} + C}$

h) $\int (9x^2 - 2x)(6x^3 - 2x^2 + 1)^3\,dx$

LET $u = 6x^3 - 2x^2 + 1$

THEN $du = (18x^2 - 4x)\,dx \Rightarrow dx = \frac{du}{2(9x^2 - 2x)}$

$\int (9x^2 - 2x)u^3 \frac{du}{2(9x^2 - 2x)} = \frac{1}{2}\int u^3\,du = \frac{1}{2}\left(\frac{1}{4}u^4\right) + C = \boxed{\frac{1}{8}(6x^3 - 2x^2 + 1)^4 + C}$

i) $\int \cos^3(2\pi x + \frac{\pi}{6})\sin(2\pi x + \frac{\pi}{6})\,dx$

LET $u = \cos(2\pi x + \frac{\pi}{6})$ THEN $du = -\sin(2\pi x + \frac{\pi}{6})(2\pi)\,dx$

$\int u^3 \sin(2\pi x + \frac{\pi}{6}) \frac{du}{-2\pi\sin(2\pi x + \frac{\pi}{6})} = \frac{-1}{2\pi}\int u^3\,du = \frac{-1}{2\pi}\left(\frac{1}{4}u^4\right) + C$

$= \boxed{\frac{-1}{8\pi}\cos^4(2\pi x + \frac{\pi}{6}) + C}$

j) $\int 3x\sin(x^2)\,dx$

LET $u = x^2$ THEN $du = 2x\,dx$

$\int 3x \sin u \frac{du}{2x} = \frac{3}{2}\int \sin u\,du = \frac{-3}{2}\cos u + C = \boxed{\frac{-3}{2}\cos(x^2) + C}$

2. a) $f''(x) = x$

$$f'(x) = \int f''(x)dx = \int x\,dx = \frac{1}{2}x^2 + C_1$$

$$f(x) = \int f'(x)dx = \int \frac{1}{2}x^2 + C_1\,dx = \frac{1}{2}\left(\frac{1}{3}x^3\right) + C_1 x + C_2$$

$$\boxed{f(x) = \frac{1}{6}x^3 + C_1 x + C_2}$$

b) $f''(x) = -9.81$

$$f'(x) = \int f''(x)dx = \int -9.81\,dx = -9.81x + C_1$$

$$f(x) = \int f'(x)dx = \int -9.81x + C_1\,dx = \boxed{\frac{-9.81}{2}x^2 + C_1 x + C_2}$$

c) $f''(x) = \sin x$

$$f'(x) = \int f''(x)dx = \int \sin x\,dx = -\cos x + C_1$$

$$f(x) = \int f'(x)dx = \int -\cos x + C_1\,dx = \boxed{-\sin x + C_1 x + C_2}$$

3. PROVE:

$$\int [f(x)g'(x) + f'(x)g(x)]\,dx = f(x)g(x) + C$$

Pf: $D_x[f(x)g(x) + C] = f'(x)g(x) + f(x)g'(x) = D_x \int [f(x)g'(x) + f'(x)g(x)]\,dx$

Q.E.D.

4. $\int |x|\,dx = ?$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}, \text{ so}$$

$$\int |x|\,dx = \begin{cases} \int x\,dx, & x \geq 0 \\ \int -x\,dx, & x < 0 \end{cases} = \boxed{\begin{cases} \frac{1}{2}x^2 + C, & x \geq 0 \\ -\frac{1}{2}x^2 + C, & x < 0 \end{cases}}$$