Math 1210-9 Exam 0

Solutions

1. Find

   (a) $\frac{2}{3} - \frac{1}{4} = \frac{8}{12} - \frac{3}{12} = \frac{5}{12}$

   (b) $\dfrac{1 + \frac{1}{3}}{\frac{1}{4} + \frac{1}{3}} = \dfrac{\frac{3}{3} + \frac{1}{3}}{\frac{3}{12} + \frac{4}{12}} = \dfrac{\frac{4}{3}}{\frac{7}{12}} = \frac{4}{3} \cdot \frac{12}{7} = \frac{16}{7}$

2. Express as a rational number

   (a) $32^{-4/5} = \frac{1}{32^{4/5}} = \frac{1}{\left(32^{1/5}\right)^4} = \frac{1}{2^4} = \frac{1}{16}$

   (b) $\frac{25^{3/2}}{64^{-5/6}} = \frac{\left(25^{1/2}\right)^3}{\left(64^{1/6}\right)^{-5}} = \frac{5^3}{2^{-5}} = \frac{125}{32^{-1}} = 125 \cdot 32 = 4000$

3. Expand

   (a) $2x(x^2+1) = 2x^3 + 2x$

   (b) $(x+y)^2 = x^2 + 2xy + y^2$

4. Factor

   (a) $x^2 - 7x + 12 = (x-4)(x-3)$

   (b) $2x^2 + 4x - 2x^3 = -2x(x^2 - x - 2) = -2x(x-2)(x+1)$

5. Perform the indicated operations and simplify

   (a)

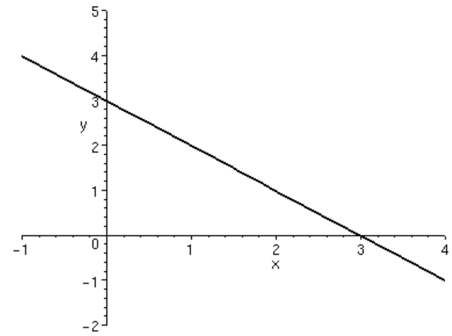
$$\begin{aligned}
\frac{x^3-8}{x^2-4} \div \frac{x^2+2x+4}{x^3+8} &= \frac{x^3-8}{x^2-4} \cdot \frac{x^3+8}{x^2+2x+4} \\
&= \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} \cdot \frac{(x+2)(x^2-2x+4)}{x^2+2x+4} \\
&= x^2 - 2x + 4
\end{aligned}$$

   (b)

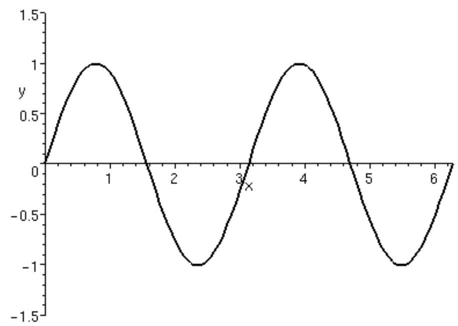
$$\begin{aligned}
\frac{3}{x-1} - \frac{2}{x} + \frac{x+3}{x^2-1} &= \frac{3x(x+1)}{x(x^2-1)} - \frac{2(x^2-1)}{x(x^2-1)} + \frac{(x+3)x}{x(x^2-1)} \\
&= \frac{3x^2 + 3x - 2x^2 + 2 + x^2 + 3x}{x(x^2-1)} \\
&= \frac{2x^2 + 6x + 2}{x(x^2-1)} \\
&= \frac{2(x^2+3x+1)}{x(x^2-1)}
\end{aligned}$$

6. Sketch the graphs of the following

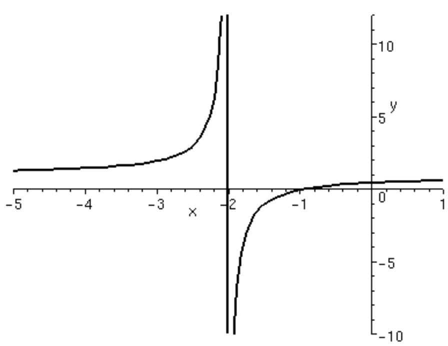
(a) $y = 3 - x$



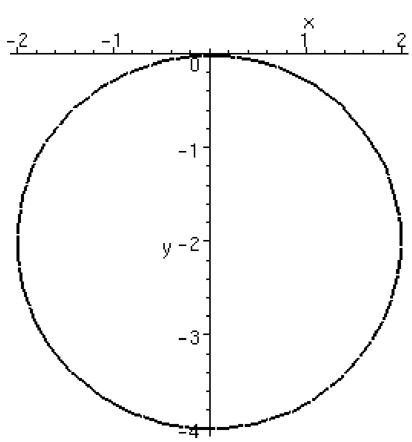
(b) $y = \sin 2x \qquad (0 \le x \le 2\pi)$



(c) $y = \dfrac{x+1}{x+2}$



(d) $x^2 + (y+2)^2 = 4$



7. Solve the equations

   (a) $2x^2 - x - 3 = 0$
   Notice that $2x^2 - x - 3 = (2x-3)(x+1)$. Hence

$$\begin{aligned} & 2x^2 - x - 3 = 0 \\ \iff & (2x-3)(x+1) = 0 \\ \iff & 2x - 3 = 0 \quad \text{or} \quad x + 1 = 0 \\ \iff & x = \frac{3}{2} \quad \text{or} \quad x = -1 \end{aligned}$$

   (b) $2x^2 + 6x + 3 = 0$
   Using the Quadratic Formula, we obtain

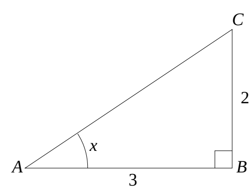
$$x = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2} = \frac{-6 \pm \sqrt{12}}{4} = \frac{-6 \pm 2\sqrt{3}}{4} = \frac{-3 \pm \sqrt{3}}{2}$$

   (c) $2x^3 - x^2 - 2x + 1 = 0$
   Since 1 is a solution, we can use scientific division and obtain:

$$2x^3 - x^2 - 2x + 1 = (x-1)(2x^2 + x - 1) = (x-1)(2x-1)(x+1)$$

   Hence the solutions are $x = 1, -1, \frac{1}{2}$.

8. Given the triangle DEF:



   What are

   (a) the length of AC?
   Using the Pythagorean Theorem, we obtain

$$|AC|^2 = |AB|^2 + |BC|^2 = 3^2 + 2^2 = 13$$

   Hence $|AC| = \sqrt{13}$.

   (b) $\cos x$?

$$\cos x = \frac{|AB|}{|AC|} = \frac{3}{\sqrt{13}}$$

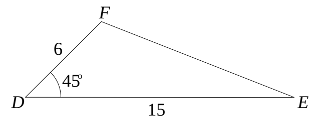
   (c) $\sin x$?

$$\sin x = \frac{|BC|}{|AC|} = \frac{2}{\sqrt{13}}$$

   (d) $\tan x$

$$\tan x = \frac{|BC|}{|AB|} = \frac{2}{3}$$

9. Given the triangle DEF:



What is the length of EF?

Using the law of cosines, we obtain

$$|EF|^2 = |DF|^2 + |DE|^2 - 2\,|DF|\,|DE|\cos(45^\circ) = 6^2 + 15^2 - 2\cdot 6\cdot 15\cdot \frac{\sqrt{2}}{2} = 261 - 90\sqrt{2}$$

Hence $|EF| = \sqrt{261 - 90\sqrt{2}} \approx 11.56377$.