Math 1060-1 Exam 3

Name: KEY

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Show all your work in a neat and organized manner. Write your answers and solutions in the space provided. Please box your answers. **You may use a calculator.**

THE POINT VALUE FOR EACH PROBLEM IS LISTED (PTS) NEXT TO EACH PROBLEM.

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(5) 1. Simplify the following trigonometric function by factoring and using trigonometric identities.

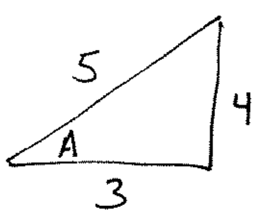
$$\sin^4 x - \cos^4 x$$

$$= (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$$

Any Of These Answers Are Acceptable

$$\boxed{\begin{aligned} &= \sin^2 x - \cos^2 x \\ &= \sin^2 x - (1 - \sin^2 x) = 2\sin^2 x - 1 \\ &= 1 - \cos^2 x - \cos^2 x = 1 - 2\cos^2 x \\ &= -\cos 2x \end{aligned}}$$

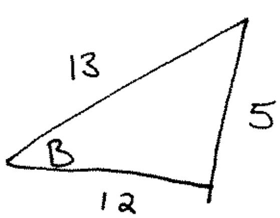
(10) 2. Given $\cos A = \frac{3}{5}$ and $\tan B = \frac{5}{12}$, and A and B lie in the first quadrant, find the following using the trig identities and without the use of a calculator:



(a) $\sin 2A$

$$= 2 \sin A \cos A$$

$$\boxed{= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)}$$



(b) $\tan(A + B)$

$$= \frac{\tan A + \tan B}{1 - \tan A \tan B} = \boxed{\frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3}\left(\frac{5}{12}\right)}}$$

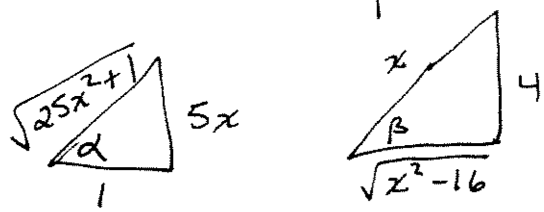
(c) $\cos\left(\frac{B}{2}\right)$

$$= \sqrt{\frac{1 + \cos B}{2}} = \boxed{\sqrt{\frac{1 + \frac{12}{13}}{2}}}$$

Make Sure The Square Root Is Over The Whole Fraction, Numerator And Denominator.

(5) 3. Write $\cos\left[\tan^{-1}(5x) + \sin^{-1}\left(\frac{4}{x}\right)\right]$ as an algebraic expression.

$\tan^{-1}(5x) = \alpha$, $\sin^{-1}\left(\frac{4}{x}\right) = \beta$



$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\boxed{= \left(\frac{1}{\sqrt{25x^2+1}}\right)\left(\frac{\sqrt{x^2-16}}{x}\right) - \left(\frac{5x}{\sqrt{25x^2+1}}\right)\left(\frac{4}{x}\right)}$$

(7) 4. Solve $\sin 5x + \sin 3x = 0$ by using a sum to product equation.

$$\sin 5x + \sin 3x = 0$$

$$2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right) = 0$$

$$2\sin(4x)\cos(x) = 0$$

$\sin 4x = 0$ $\qquad$ $\cos x = 0$

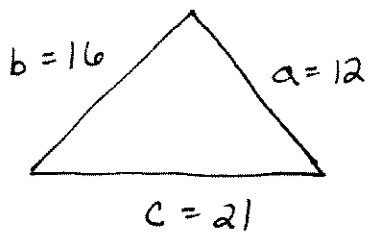
$4x = \pi n$ $\qquad$ $\boxed{x = \frac{\pi}{2} + \pi n}$

$\boxed{x = \frac{\pi n}{4}}$

THEY MUST HAVE ALL SOLUTIONS, NOT JUST FROM 0 TO 2π

(8) 5. Farmer Joe has some prefabricated fence pieces which come in three lengths, $a = 12$ ft, $b = 16$ ft, and $c = 21$ ft. He wants to enclose a triangular pasture with these pieces of fence.

(5) (a) At what angles, A, B and C, should the pieces be placed?



ALWAYS SOLVE FOR THE LARGEST ANGLE FIRST:

$$21^2 = 16^2 + 12^2 - 2(16)(12)\cos C$$

$$\boxed{C \approx 96.13^\circ}$$

THEN WE CAN USE THE LAW OF SINES:

$$\frac{\sin 96.13^\circ}{21} = \frac{\sin A}{12} \qquad \boxed{A \approx 34.62^\circ}$$

$$B \approx 180^\circ - C - A = \boxed{49.25^\circ}$$

(3) (b) What size of pasture will he enclose? (i.e., area.)

$$S = \frac{a+b+c}{2} = 24.5$$

$$\text{AREA} = \sqrt{24.5(3.5)(8.5)(12.5)} \approx \boxed{95.45 \text{ ft}^2}$$

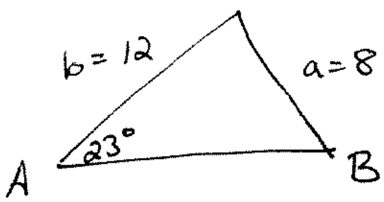
OR

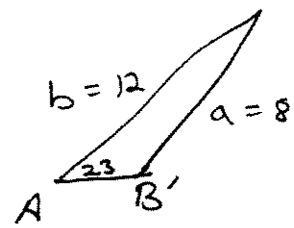
$$\text{AREA} = \frac{1}{2}(16)(21)\sin(34.62^\circ) \approx \boxed{95.45 \text{ ft}^2}$$

(5) 6. Suppose you are given a triangle with the following measurements:

$$A = 23^\circ, b = 12, a = 8$$

There are two possible triangles: one with an acute angle B and one with an obtuse angle B'. Find B and B'.





$$\frac{\sin 23^\circ}{8} = \frac{\sin B}{12}$$

$$\boxed{B \approx 35.88^\circ}$$

$$B' = 180 - B$$

$$\boxed{B' = 144.12^\circ}$$

(5) 7. (Extra Credit) What substitution should you make to write $\sqrt{4 - x^2}$ as a trigonometric function of θ.

$\boxed{x = 2\sin\theta}$ OR $\boxed{x = 2\cos\theta}$

BECAUSE:

$$\sqrt{4 - (2\sin\theta)^2}$$
$$= \sqrt{4 - 4\sin^2\theta}$$
$$= \sqrt{4(1 - \sin^2\theta)}$$
$$= \sqrt{4\cos^2\theta}$$
$$= 2\cos\theta$$

$$\sqrt{4 - (2\cos\theta)^2}$$
$$= \sqrt{4 - 4\cos^2\theta}$$
$$= \sqrt{4(1 - \cos^2\theta)}$$
$$= \sqrt{4\sin^2\theta}$$
$$= 2\sin\theta$$