Math 1060-1 Exam 2

Name:______________________________

ID# ______________________________

Show all your work in a neat and organized manner. Write your answers and solutions in the space provided. Please box your answers. You may use a calculator and a 3x5 notecard.

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1. (5 pts) Simplify the following trigonometric function using trigonometric identities.

$$(1 - \sin^2 x)(\sec x)$$

$$= \cos^2 x \left(\frac{1}{\cos x}\right) = \boxed{\cos x}$$

2. (10 pts) Using a double angle formula, solve the following equation on the interval $[0, 2\pi)$.

$$\sin(2x) - \sin x = 0$$

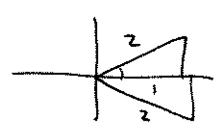
$$2 \sin x \cos x - \sin x = 0$$

$$\sin x (2 \cos x - 1) = 0$$

$\sin x = 0$ $\qquad$ $2 \cos x - 1 = 0$

$\boxed{x = 0, \pi}$ $\qquad$ $2 \cos x = 1$

$$\cos x = \frac{1}{2}$$



$$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$$

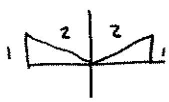
3. (10 pts) The monthly sales (in thousands of units) of a seasonal product are approximated by

$$S = 73 + 54\sin\left(\frac{\pi t}{6}\right)$$

where t is the time in months, with $t = 1$ corresponding to January. Determine the months when sales are 100,000, i.e., when $S = 100$.

$$100 = 73 + 54\sin\left(\frac{\pi t}{6}\right)$$

$$\frac{1}{2} = \sin\left(\frac{\pi t}{6}\right)$$



$$\frac{\pi t}{6} = \frac{\pi}{6} + 2\pi n \qquad\qquad \frac{\pi t}{6} = \frac{5\pi}{6} + 2\pi n$$

$$t = 1 + 12n \qquad\qquad t = 5 + 12n$$

$$\boxed{\text{JANUARY}} \qquad\qquad \boxed{\text{MAY}}$$

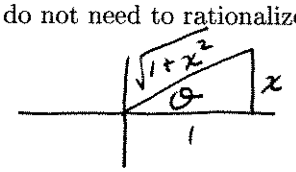
4. (5 pts) Use the substitution $x = 3\cos\theta$ to write $\sqrt{9 - x^2}$ as a trigonometric function of θ.

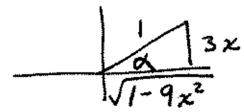
$$\sqrt{9 - (3\cos\theta)^2} = \sqrt{9 - 9\cos^2\theta}$$

$$= \sqrt{9(1 - \cos^2\theta)} = \sqrt{9\sin^2\theta}$$

$$\boxed{= 3\sin\theta}$$

5. (10 pts) Write $\cos(\arctan x - \arcsin 3x)$ as an algebraic expression. (You do not need to rationalize the denominator in the answer.)





$\arctan x = \theta$ $\qquad$ $\arcsin 3x = \alpha$

$$\cos(\theta - \alpha) = \cos\theta \cos\alpha + \sin\theta \sin\alpha$$

$$= \frac{1}{\sqrt{1+x^2}} \sqrt{1-9x^2} + \frac{x}{\sqrt{1+x^2}} 3x$$

$$\boxed{= \frac{\sqrt{1-9x^2} + 3x^2}{\sqrt{1+x^2}}}$$

6. (5 pts) Write $3 \sin 2\alpha \sin 3\alpha$ as a sum or difference.

$$3 \sin 2\alpha \sin 3\alpha$$

$$= 3\left[\frac{1}{2}\left(\cos(2\alpha - 3\alpha) - \cos(2\alpha + 3\alpha)\right)\right]$$

$$= \frac{3}{2}\left(\cos(-\alpha) - \cos(5\alpha)\right)$$

$$\boxed{= \frac{3}{2}\left[\cos\alpha - \cos(5\alpha)\right]}$$

7. (5 pts) Use a half-angle formula to determine the exact value of $\cos\left(\frac{\pi}{8}\right)$.

$$\cos\left(\frac{\pi}{8}\right) = \cos\left(\frac{\pi/4}{2}\right) = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}$$

$$= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$$

$$\boxed{= \frac{\sqrt{2+\sqrt{2}}}{2}}$$

8. (Extra Credit, 5 pts) Use a power-reducing formula to rewrite $\cos^4 x$ in terms of the first power of the cosine.

$$\cos^4 x = (\cos^2 x)^2 = \left(\frac{1+\cos 2x}{2}\right)^2$$

$$= \frac{1}{4}\left[1 + 2\cos 2x + \cos^2 2x\right]$$

$$\boxed{= \frac{1}{4}\left[1 + 2\cos 2x + \frac{1+\cos 4x}{2}\right]}$$