MATH 6610
Fall 2016

Homework 4, Due November 11 2016

Show all the work. Please submit your working codes via e-mail. Late homework will not be accepted.

Problem 1 (*Computational Assignment*).
Trefethen, page 101, problem 13.3. Give conclusions on what you see.
Problem 2.
a) Assume that the matrix norm $||\cdot||$ satisfies the submultiplicative property $||AB|| \leq ||A||||B||$. Show that if $||X|| < 1$, then: 1) $I - X$ is invertible, 2) $(I-X)^{-1} = \sum_{j=0}^{\infty} X^j$, and 3) $||(I-X)^{-1}|| \leq 1/(1-||X||)$.
b) Let $A \in R^{n\times n}$ be nonsingular matrix. Assume that $b \neq 0$, x satisfies $Ax = b$, and $\tilde{x}$ is an approximate solution to this linear system. Denote $e := x - \tilde{x}$ - the error vector and $r := b - A\tilde{x}$ - the residual vector.
Show the following inequalities, and explain their importance.

$$\frac{1}{||A||||A^{-1}||}\frac{||r||}{||b||} \leq \frac{||e||}{||x||} \leq ||A||||A^{-1}||\frac{||r||}{||b||}$$

c) Suppose that $A \in R^{n\times n}$ is nonsingular, and consider the two problems:

$$Ax = b$$

and

$$(A + \delta A)\tilde{x} = b + \delta b,$$

where we assume that $||A^{-1}\delta A|| \leq ||A^{-1}|| \cdot ||\delta A|| < 1$, so that $(A + \delta A)$ is nonsingular (Why?)
Show that

$$\frac{||\tilde{x} - x||}{||x||} \leq \frac{\kappa(A)}{1 - \kappa(A)\frac{||\delta A||}{||A||}}\Big(\frac{||\delta A||}{||A||} + \frac{||\delta b||}{||b||}\Big),$$

$\kappa(A)$ is the condition number of the matrix.
Problem 3.
Show that for Gaussian elimination with partial pivoting (permutation by rows) applied to a matrix $A \in R^{n\times n}$, the growth factor $\rho = \max_{ij}|u_{ij}|/\max_{ij}|a_{ij}|$ satisfies the estimate $\rho \leq 2^{n-1}$.
Problem 4.
Show that if all the principal minors of a matrix $A \in R^{n\times n}$ are nonzero, then there exist diagonal matrix D, unit lower triangular matrix L and unit upper triangular matrix U, such that $A = LDU$. Is this factorization unique? What happens if A is symmetric matrix?
Problem 5 (*Computational Assignment*).
Implement Cholesky method to solve linear system $Ax = b$, where $A \in R^{n\times n}$ is spd. Suggest three test problems and test your code and the algorithm on the selected test problems (suggest examples of the linear system $Ax = b$, where A is spd. Compare the results with matlab routine "chol"). Discuss the results.