

Homework 3, Due October 21 2016

Show all the work. Please submit your working codes via e-mail. Late homework will not be accepted.

Problem 1.

a) Let  $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 1 & 1 & -3 \\ 0 & 1 & 1 \end{pmatrix}$  and  $b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

1. Determine the reduced  $QR$  factorization of  $A$
  2. Use the  $QR$  factors from part 1 to determine the least squares solution to  $Ax = b$
- b) Trefethen page 55, Problem 7.5

Problem 2.

Trefethen page 55, Problem 7.3

Problem 3.

Consider the inner-product space of real-valued continuous functions defined on  $[-1, 1]$ , where the inner product is defined by

$$f \cdot g = \int_{-1}^1 f(x)g(x)dx.$$

Let  $M$  be the subspace that is spanned by the three linearly independent polynomials  $p_0 = 1, p_1 = x, p_2 = x^2$ .

1. Use the Gram-Schmidt process to determine an orthonormal set of polynomials (Legendre polynomials)  $q_0, q_1, q_2$  that spans  $M$ .
2. Check that  $q_n$  satisfies  $(1 - x^2)y'' - 2xy' + n(n + 1)y = 0$  for  $n = 0, 1, 2$

Problem 4. (*Computational Assignment*)

Trefethen page 68, Problem 9.3

Problem 5.

a) Let  $A \in \mathbb{R}^{m \times n}$  with  $m < n$  and of full rank. Then  $\min \|Ax - b\|_2$  is called an Underdetermined Least-Squares Problem. Show that the solution is an  $n - m$  dimensional set. Show how to compute the unique minimum norm solution using QR decomposition and SVD approach.

b) (*Computational Assignment*) Trefethen page 85, Problem 11.3 (a, b, e, f).

Your calculations will produce lists of twelve coefficients. In each list highlight the digits that appear to be wrong (affected by rounding error). Comment on what differences you observe. Do the normal equations exhibit instability?

c) (*For extra bonus points of 30 points total*) *Bonus Computational Question (not required):*

Trefethen page 85, Problem 11.3 (c, d).

Your calculations will produce lists of twelve coefficients. In each list highlight the digits that appear to be wrong (affected by rounding error). Comment on what differences you observe.